Math 1310
Section 5.4: Properties of Logarithms

You will sometimes be asked to rewrite logarithmic expressions in either an expanded or contracted form. To do this, you will use the Laws of Logarithms. You must know all 8 of the Laws of Logarithms.

Laws of Logarithms
If m, n and a are positive numbers, $a \neq 1$, then

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n\log_a m$
4. $\log_a 1 = 0$
5. $\log_a a = 1$
6. $\log_a a^x = x$
7. $a^{\log_a x} = x$
8. $\log_a m = \frac{\log m}{\log a}$ (change of bases formula)

These properties are true for logs of any base, including common logs and natural logs.

Example 1: Rewrite each of the following expressions in a form that has no logarithms of a product, quotient or power.

a. $\log\left(\frac{3x}{y}\right)$

b. $\ln(a^3bc)$

c. $\log_4\left(\frac{4}{x}\right)$

d. $\log\sqrt{xy}$

e. $\ln\left(\frac{\sqrt[3]{x+4}}{(x+2)^4(x-5)^3}\right)$

f. $\log\sqrt{\frac{(x-1)(x+2)^3}{x^2(x-2)}}$

Example 2: Express each as a single logarithm:

a. $\log x - 3\log y$

b. $2\ln x + 3\ln(x + 2) - 5\ln y$

c. $\frac{1}{2}\ln(x+2) - 3\ln(x^3+1)$

d. $-2\log_4(x-5) - \log_4(x+1) + 2\log_4(x^2+1)$

e. $\ln 18 - \ln 2$

Example 3: Rewrite each as sums so that each logarithm contains a prime number.

a. $\ln 72$

b. $\log_2 96$

Example 4: Use the change of bases formula to solve $\log_8 12 = x$ and write in simplest form.

Example 5: Simplify each.

a. $\log_4 16^3$

b. $7^{3\log_7 3}$