Section 6.1/6.2*

2x2 Linear Systems and some other Systems/Applications

Solving 2x2 Linear Systems

To solve a system of two linear equations $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$ means to find values for x and y that satisfy both equations. You may use either the substitution or elimination/addition method. A 2x2 linear system of equations may have no solution, one solution or infinitely many solutions.

Example 1: Solve each system of linear equations by **the Substitution Method.**

a. $2x - y = 5$
   $5x + 2y = 8$

b. $x - 2y = 3$
   $2x - 4y = 7$

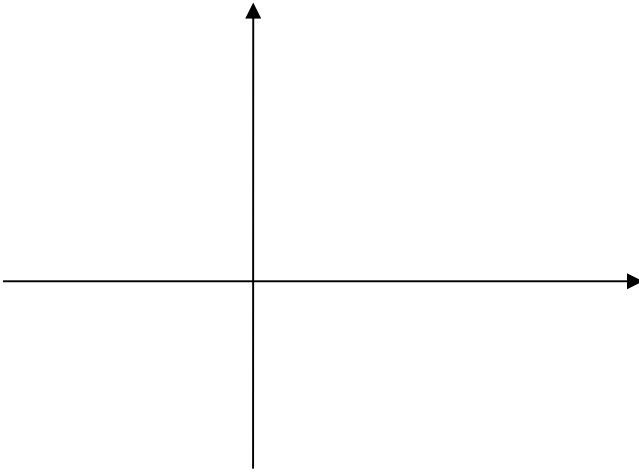**Example 2:** Solve the following systems by **the Elimination Method**.

a. $x + 4y = 10$

$\dfrac{1}{2}x + 2y = 5$

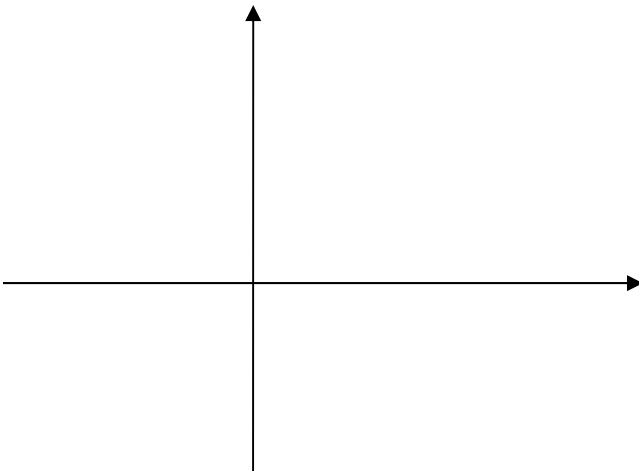b. $y - 2x = -6$

$2y - x = 0$

c. $2x + 3y = -16$

$5x - 10y = 30$

A system will have exactly one solution, no solution, or infinitely many solutions.
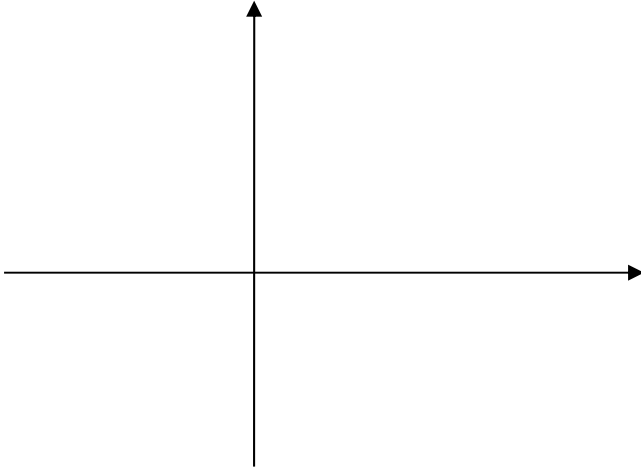
1. Exactly one solution, will look like:

2. No solution, will look like:

3. Infinitely many solutions, will look like:

If you are interested in determining the number of solutions a 2X2 linear system of equations has, you can use the slope and y-intercept to determine just that.

Parallel and Perpendicular Lines

Two lines are parallel if and only if their slopes are the same.

Two lines are perpendicular if and only if their slopes are negative reciprocals of each other.

Example 3: Determine the number of solutions each of the following system has by using the slope and y-intercept.

a.
$$y = \frac{3}{5}x$$
$$y = -\frac{5}{3}x + 1$$

b.
$$y = -3x - 1$$
$$5y = -15x + 2$$

c. $\begin{array}{l} x - 2y = 4 \\ -3x + 6y = -12 \end{array}$

d. $\begin{array}{l} y = -3x + 5 \\ x - 2y = 6 \end{array}$

Example 4. Solve the following systems:

a. $\begin{array}{l} y = x^2 \\ x + y = 12 \end{array}$

b. $\begin{array}{l} x + y^2 = 5 \\ x + y = 3 \end{array}$

For each of the following problems:

(a) Write a system of equations involving two variables to model the problem.
(b) Solve your system of equations and answer the question.

Example 1. Dillan is at a baseball game and is buying hot dogs and sodas for his family. Hot dogs cost $3 each and sodas cost $1.75 each. He purchases nine items and spends a total of $22.00. How many hot dogs did he buy? How many sodas did he buy?

Example 2. Two numbers have a sum of 77 and a difference of 13. Find the two numbers

Example 3. A rectangle has a perimeter of 26 centimeters and an area of 36 square centimeters. Find the dimensions of the rectangle.