Math 1330 Section 5.2

For the following functions:

$$y = A\sin(Bx - C) \quad \text{and} \quad y = A\cos(Bx - C)$$

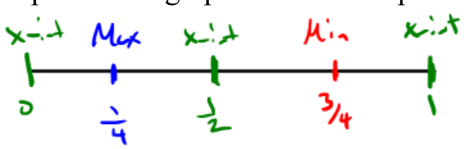
Amplitude = $|A|$ (Note: Amplitude is always positive.) If A is negative, that means an *x*- axis reflection.

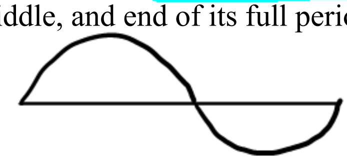
Period = $\dfrac{2\pi}{B}$ $B = \dfrac{2\pi}{\text{Period}}$

Translation in horizontal direction (called the *phase shift*) = $\dfrac{C}{B}$

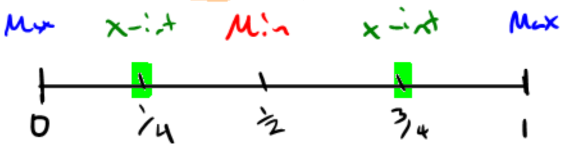
We'll ask you to learn the shape of the graph and just graph five basic points, the *x* and *y* intercepts and the maximum and the minimum.

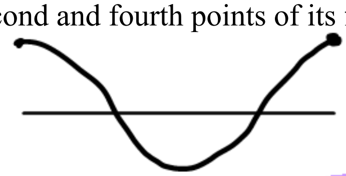
One complete cycle of the sine curve includes three *x*-intercepts, one maximum point and one minimum point. The graph has *x*-intercepts at the beginning, middle, and end of its full period.





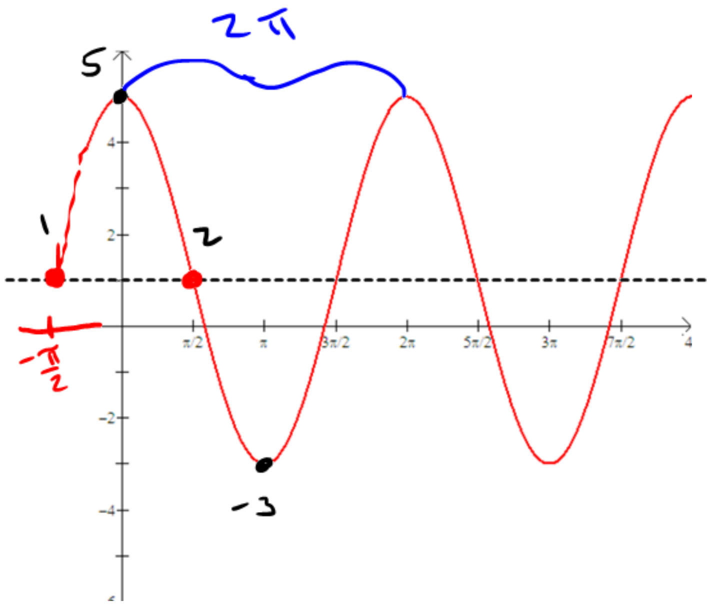
One complete cycle of the cosine curve includes two *x*-intercepts, two maximum points and one minimum point. The graph has *x*-intercepts at the second and fourth points of its full period.





Key points in graphing these functions are obtained by dividing the period into four equal parts.

Example 9: Give a function of the form $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$, which could be used to represent the graph. *Note:* these answers are not unique.



Amp: $\frac{5-(-3)}{2} = \frac{8}{2} = 4$

$A = 4$

Vertical Shift (D)

$D = \frac{Max + Min}{2} = \frac{5 + (-3)}{2} = 1$

Period $= 2\pi$

$B = \frac{2\pi}{Period} = \frac{2\pi}{2\pi} = 1$

For cos(x)

No phase shift; $C = 0$

$y = 4\cos(1 \cdot x) + 1$

$y = 4\cos(x) + 1$

For sin x; Need a phase shift

① $y = 4\sin(1 \cdot x - C) + 1$

Back $\frac{\pi}{2}$ units

$\frac{\pi}{2} = \frac{C}{B}$; $C = \frac{\pi}{2} \cdot B$

$C = \frac{\pi}{2}$

$y = 4\sin\left(x + \frac{\pi}{2}\right) + 1$

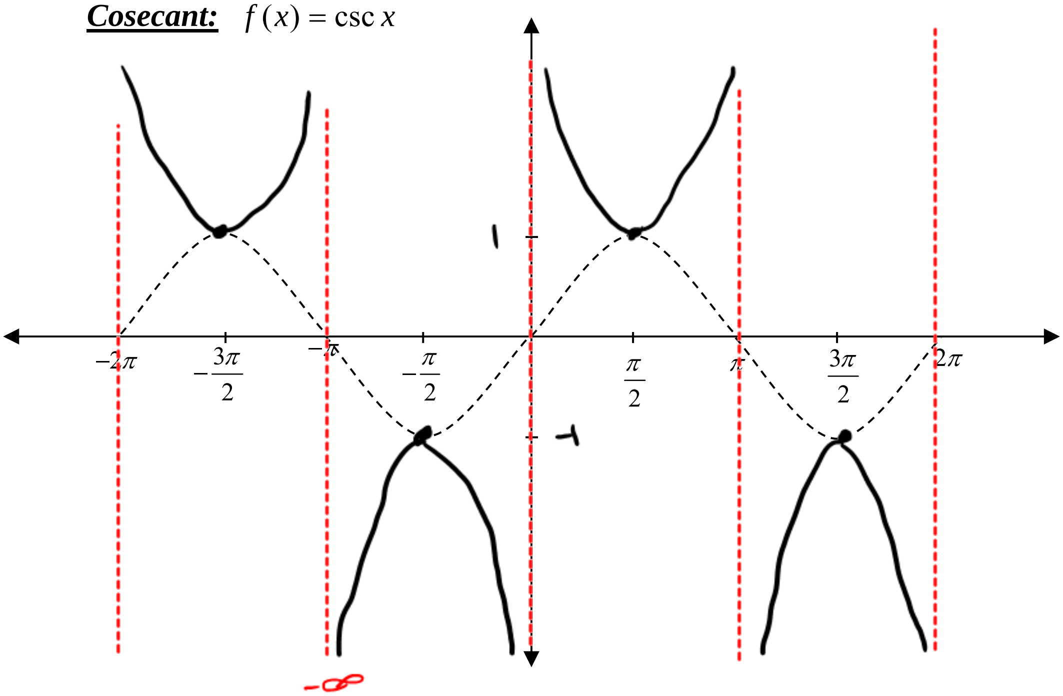
or

② $y = -4\sin\left(x - \frac{\pi}{2}\right) + 1$

Section 5.3a: Secant and Cosecant Functions

Remember: $\csc x = \dfrac{1}{\sin x}$ so whenever $\sin(x) = 0, \csc(x)$ has an asymptote.

Cosecant: $f(x) = \csc x$



Domain: $x \neq n\cdot\pi$

Range: $(-\infty,-1] \cup [1,\infty)$

Period: 2π

Vertical Asymptotes:

$n\cdot\pi$

x- intercepts:

None

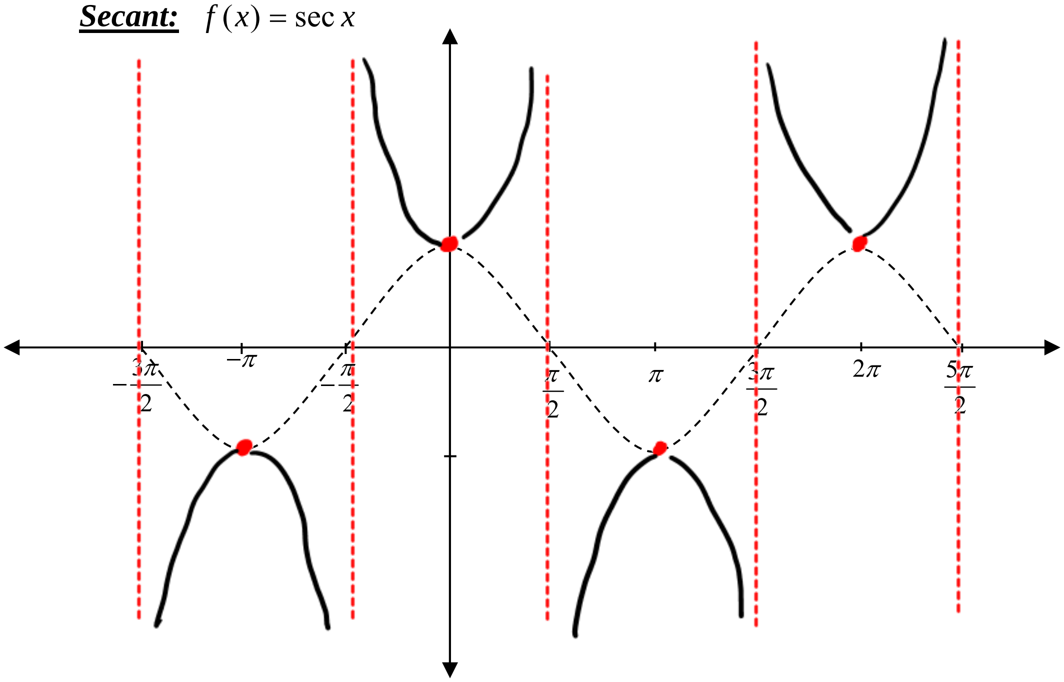
y- intercept: None

To graph $y = A\csc(Bx - C)$, first graph, the helper graph, $y = A\sin(Bx - C)$

Remember: $\sec x = \dfrac{1}{\cos x}$ so whenever $\cos(x) = 0, \sec(x)$ has an asymptote.

$x \neq \frac{(2n-1)\pi}{2}$

Secant: $f(x) = \sec x$



Domain: $x \neq \frac{n\cdot\pi}{2}$ n is odd

Range: $(-\infty,-1] \cup [1,\infty)$

Period: 2π

Vertical Asymptotes:

$x = \frac{n\cdot\pi}{2}$ or $\frac{(2n-1)\pi}{2}$

*n is odd

x- intercepts:

None

y- intercept: $(0, 1)$

To graph $y = A\sec(Bx - C)$, first graph $y = A\cos(Bx - C)$.

Math 1330 Section 5.3a

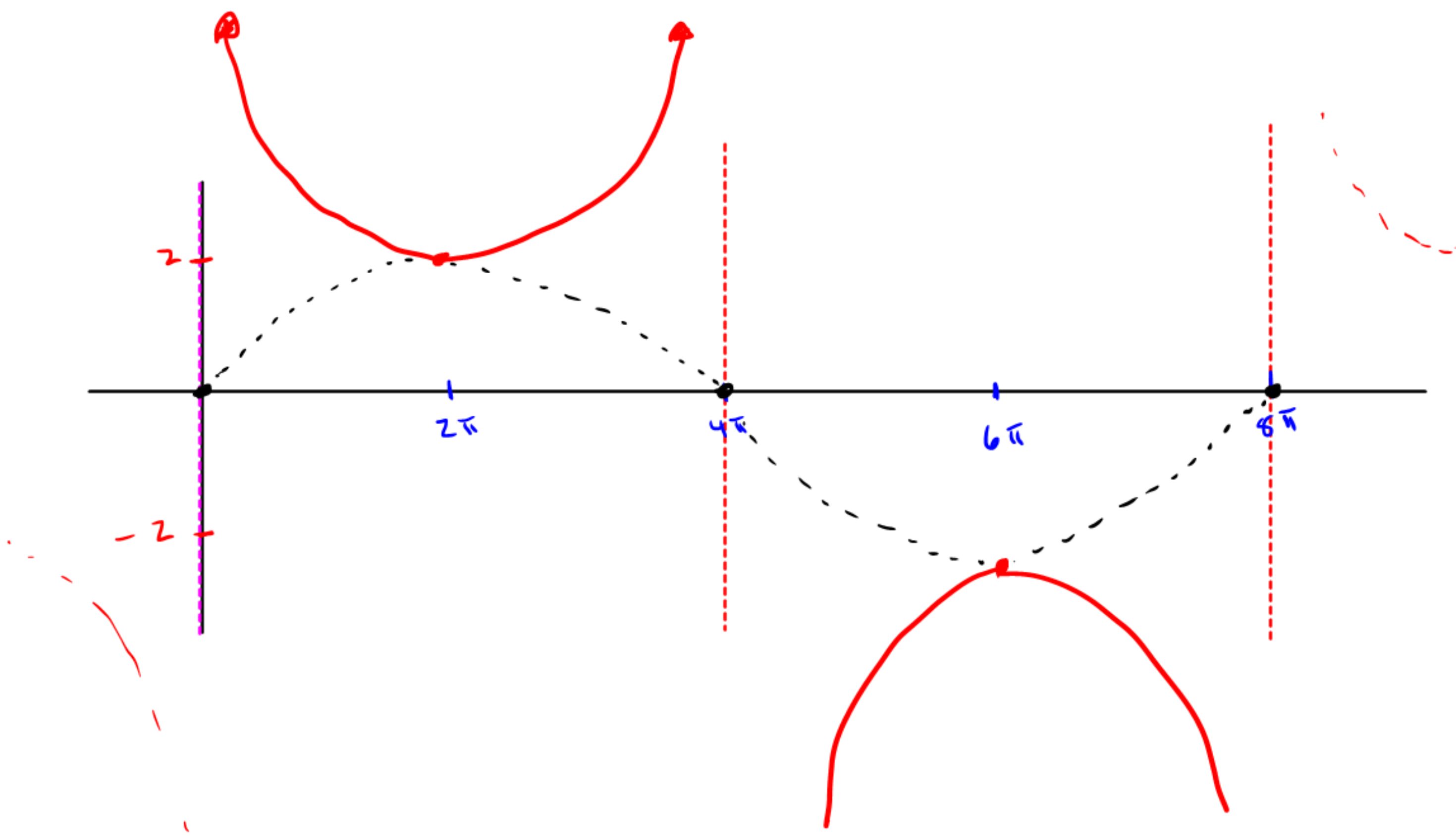
Example 1: Graph $y = 2\csc\left(\frac{x}{4}\right)$ over one period.

Helper function

$y = \underline{2} \sin\left(\frac{x}{4}\right) \rightarrow$ 3 x int, 1 Max, 1 Min

Amp: 2

Period: $\frac{2\pi}{B} = \frac{2\pi}{1/4} = 8\pi$



Math 1330 Section 5.3a

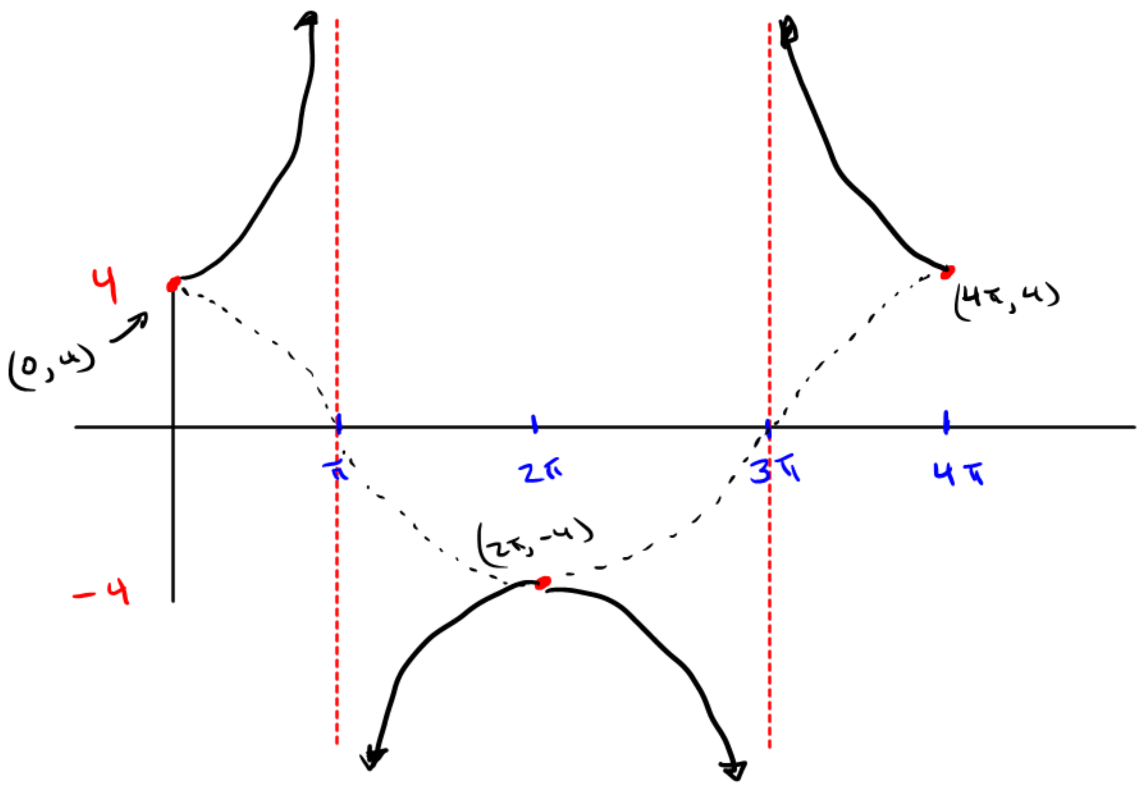
Example 2: Graph $f(x) = 4\sec\left(\frac{x}{2}\right)$ over one period.

Helper $y = 4\cos\left(\frac{x}{2}\right)$

Amp $= 4$

Period $= \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

2 x-ints
2 Max
1 Min



Math 1330 Section 5.3a

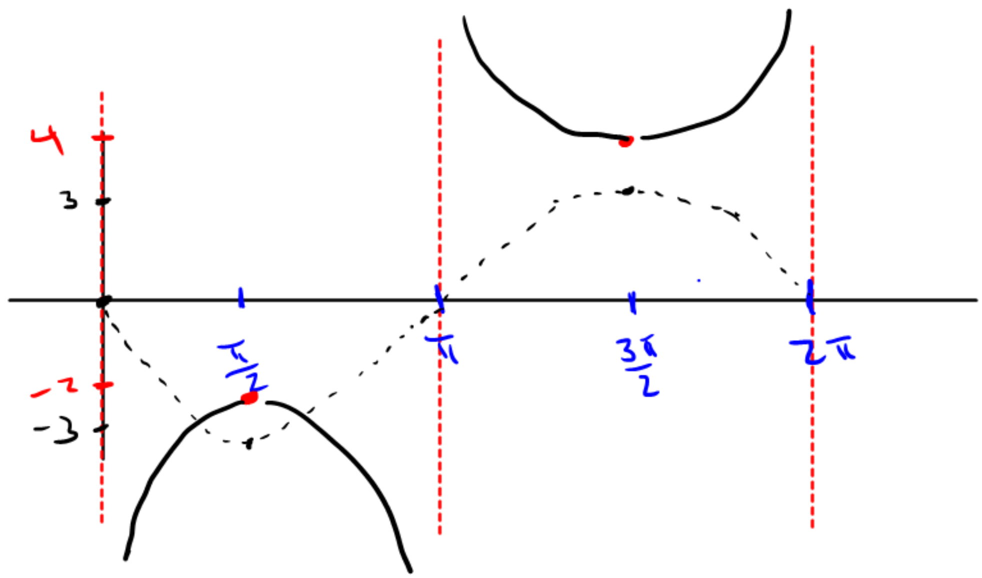
Example 3: Sketch $f(x) = -3\csc(x) + 1$ over one period.

up 1

$y = -3\sin(x) \cancel{+1}$

Amp = 3

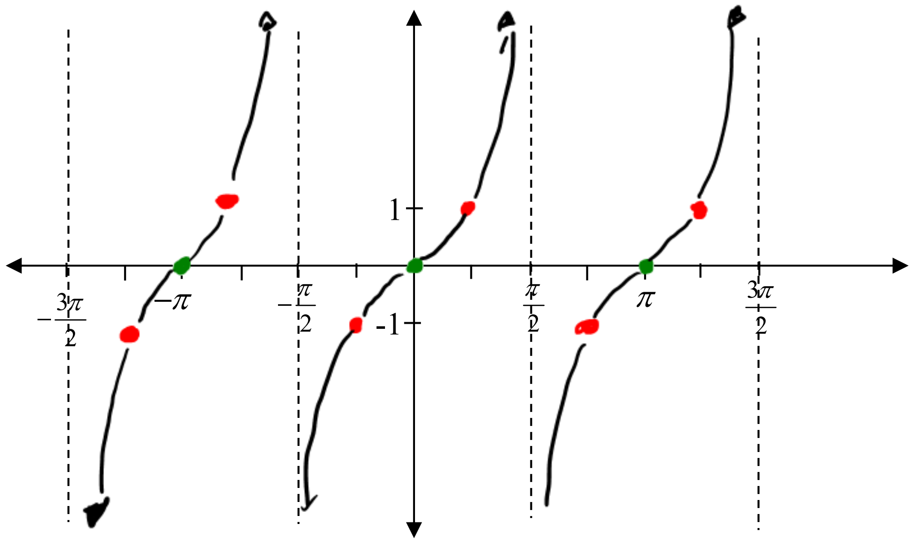
Period $= \frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$



Section 5.3b: Graphs of the Tangent and Cotangent Functions

Remember $\tan x = \dfrac{\sin x}{\cos x}$, so where $\cos(x) = 0$, $\tan(x)$ has an asymptote and where $\sin(x) = 0$, $\tan(x)$ has an x-intercept.

Tangent: $f(x) = \tan x$



Domain: $x \neq \frac{(2n-1)\pi}{2}$

Range: $(-\infty, \infty)$

Period: π

Vertical Asymptotes:

$x = \frac{(2n-1)}{2} \cdot \pi$

x- intercepts:

$n \cdot \pi$

y- intercept: $(0,0)$

How to graph $y = A\,tan(Bx - C)$:

1. The period is given by $\dfrac{\pi}{B}$. Find two consecutive asymptotes by setting $Bx - C$ equal to $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$, then solve for x.

2. Find the x-intercept by taking the average of the two points on the x-axis where consecutive asymptotes pass.

3. Find the points on the graph ¼ and ¾ of the way between the consecutive asymptotes. The y-coordinates of these points are $-A$ and A

Math 1330 Section 5.3b

Example 1: Graph $f(x) = 1\tan\left(\frac{x}{2} - \frac{\pi}{4}\right)$ over one period.

$A = 1$

Find Asymptotes

Left $Bx - C = -\frac{\pi}{2}$

$\frac{x}{2} - \frac{\pi}{4} = -\frac{\pi}{2}$

$\frac{x}{2} = -\frac{\pi}{4}$

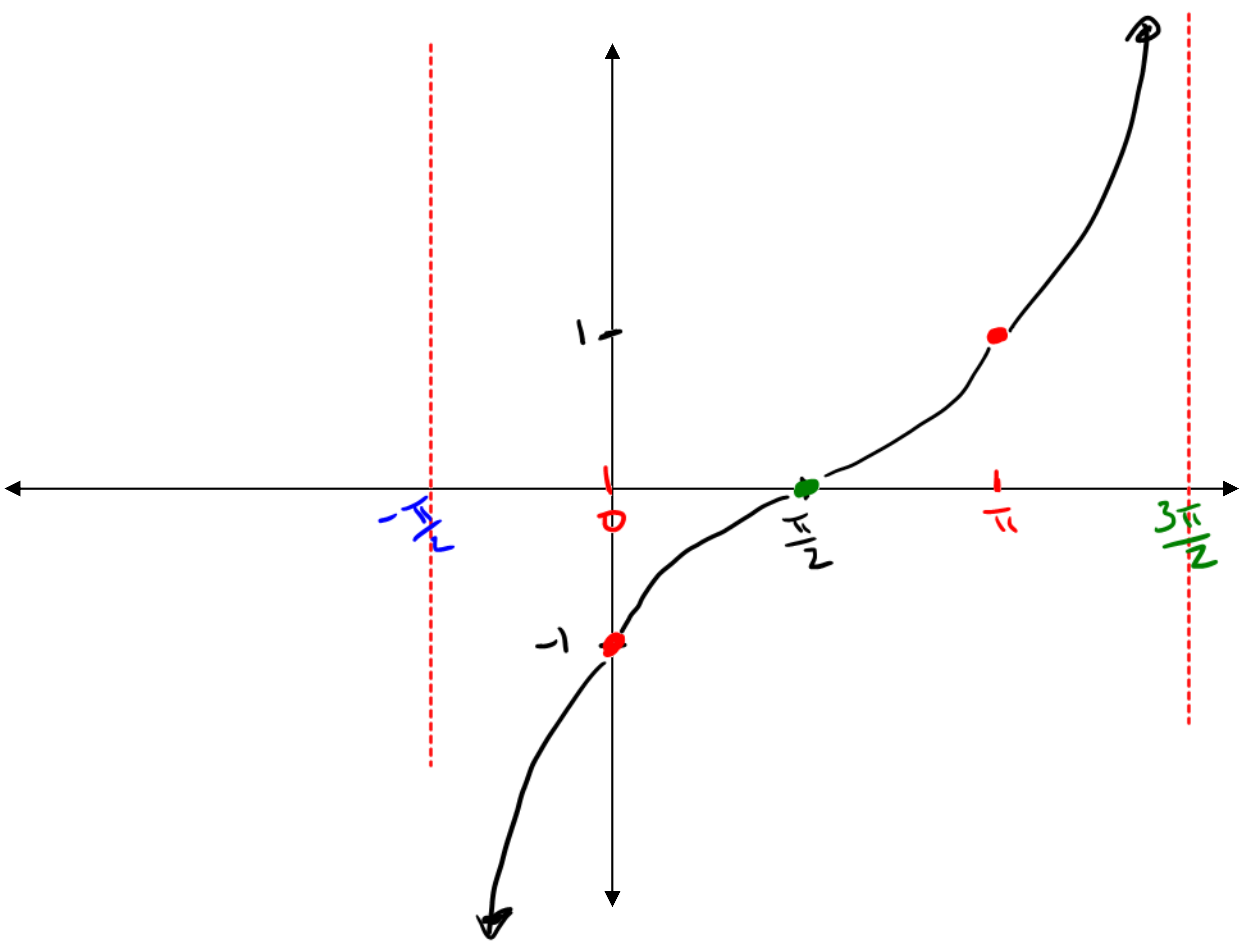
$x = -\frac{2\pi}{4} = -\frac{\pi}{2}$

Right $Bx - C = \frac{\pi}{2}$

$\frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{2}$

$\frac{x}{2} = \frac{3\pi}{4}$

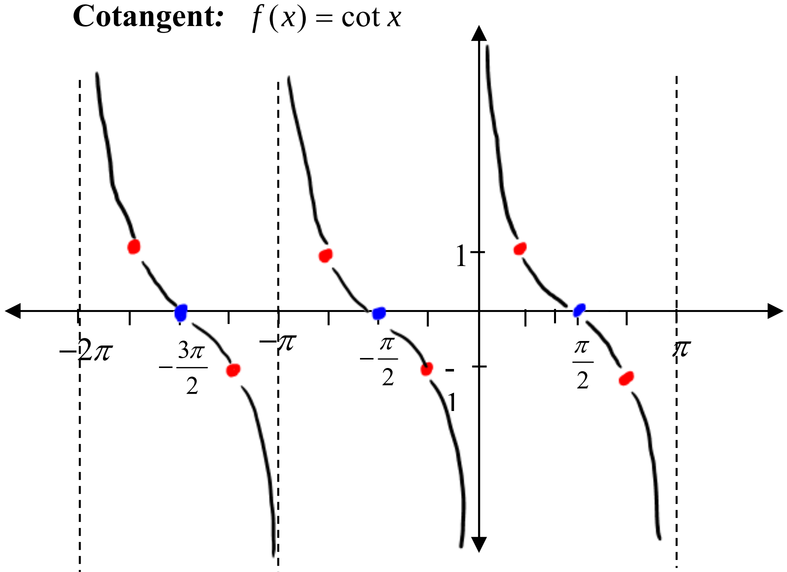
$x = \frac{6\pi}{4} = \frac{3\pi}{2}$



Math 1330 Section 5.3b

Remember $\cot x = \dfrac{\cos x}{\sin x}$, so where $\sin(x) = 0$, ~~tan~~ cot(x) has an asymptote and where $\cos(x) = 0$, ~~tan~~ cot(x) has an x-intercept.

Cotangent: $f(x) = \cot x$



Domain: $x \neq n \cdot \pi$

Range: $(-\infty, \infty)$

Period: π

Vertical Asymptotes:

$x = n \cdot \pi$

x- intercepts:

$x = \dfrac{(2n-1)\pi}{2}$ or $\dfrac{\text{odd} \cdot \pi}{2}$

y- intercept: None

How to graph $y = A\cot(Bx - C)$:

1. The period is given by $\dfrac{\pi}{B}$. Find two consecutive asymptotes by setting $Bx - C$ equal to 0 and π, then solve for x.

2. Find the x-intercept by taking the average of the two points on the x-axis where consecutive asymptotes pass.

3. Find the points on the graph ¼ and ¾ of the way between the consecutive asymptotes. The y-coordinates of these points are $-A$ and A

Math 1330 Section 5.3b

Example 2: Graph $y = \cot 2x$ over one period.

$$\text{Period} = \frac{\pi}{B} = \frac{\pi}{2} = \frac{\pi}{2}$$

Left Asymptote

$Bx - C = 0$

$2x = 0$

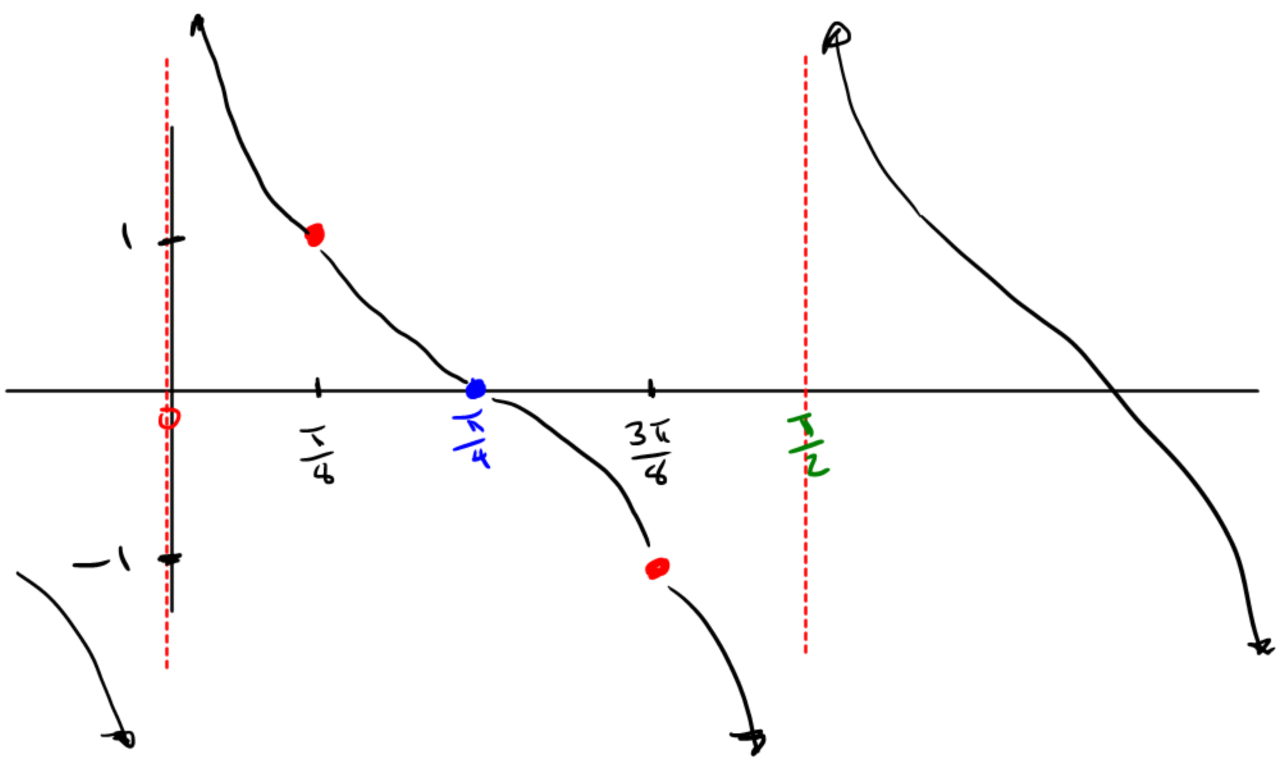
$x = 0$

Right Asymptote

$Bx - C = \pi$

$2x = \pi$

$x = \frac{\pi}{2}$



Math 1330 Section 5.3b

Example 3: Sketch $f(x) = \cot\left(x - \frac{\pi}{2}\right) - 2$

$\text{Period} = \frac{\pi}{B} = \frac{\pi}{1} = \pi$

L.A.

$Bx - C = 0$

$x - \frac{\pi}{2} = 0$

$x = \frac{\pi}{2}$

R.A.

$Bx - C = \pi$

$x - \frac{\pi}{2} = \pi$

$x = \frac{3\pi}{2}$