**Math 1313**
**Test 2 Supplemental Review Solutions**

1. a. C(x) = 6x + 180,000
b. R(x) = 12x
c. P(x) = 6x – 180,000
d. P(28,000) = -12,000; Loss of $12,000
e. P(50,000) = $120,000; Profit of $120,000
f. x = 30,000 units
g. R(30,000) = $360,000
h. (30,000, 360,000)

2. a. Slope is -14,000, so the rate of depreciation is $14,000.
V(t) = -14,000t + 60,000
b. V(3) = $18,000

3. $\left(\begin{array}{ccc|c} 8 & -4 & -2 & -10 \\ 0 & 7 & -1/2 & 0 \\ 2 & 0 & -8 & -5 \end{array}\right)$

4. $-9x + 3y + \frac{8}{7}z = -1$
$5x + 8y = 0$
$5x + 3y - z = 4$

5. a. Yes; b. No; c. Yes; d. No; e. No; f. Yes; g. Yes; h. Yes

6. a.

$$\left(\begin{array}{cc|c} 5 & 3 & 9 \\ -2 & 1 & -8 \end{array}\right) \xrightarrow{\frac{1}{5}R_1} \left(\begin{array}{cc|c} 1 & 3/5 & 9/5 \\ -2 & 1 & -8 \end{array}\right) \xrightarrow{2R_1+R_2} \left(\begin{array}{cc|c} 1 & 3/5 & 9/5 \\ 0 & 11/5 & -22/5 \end{array}\right) \xrightarrow{\frac{5}{11}R_2} \left(\begin{array}{cc|c} 1 & 3/5 & 9/5 \\ 0 & 1 & -2 \end{array}\right)$$

$$\xrightarrow{\frac{-3}{5}R_1+R_2} \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right) \quad x = 3, y = -2 \text{ or } (3, -2).$$

b.

$$\left(\begin{array}{ccc|c}2&3&-6&-11\\1&-2&3&9\\3&1&0&7\end{array}\right)\xrightarrow{R_1\leftrightarrow R_2}\left(\begin{array}{ccc|c}1&-2&3&9\\2&3&-6&-11\\3&1&0&7\end{array}\right)\xrightarrow{-2R_1+R_2;-3R_1+R_3}\left(\begin{array}{ccc|c}1&-2&3&9\\0&7&-12&-29\\0&7&-9&-20\end{array}\right)$$

$$\xrightarrow{\frac{R_2}{7}}\left(\begin{array}{ccc|c}1&-2&3&9\\0&1&-\frac{12}{7}&-\frac{29}{7}\\0&7&-9&-20\end{array}\right)\xrightarrow{2R_2+R_1;-7R_2+R_3}\left(\begin{array}{ccc|c}1&0&-\frac{3}{7}&\frac{5}{7}\\0&1&-\frac{12}{7}&-\frac{29}{7}\\0&0&3&9\end{array}\right)\xrightarrow{\frac{R_3}{3}}\left(\begin{array}{ccc|c}1&0&-\frac{3}{7}&\frac{5}{7}\\0&1&-\frac{12}{7}&-\frac{29}{7}\\0&0&1&3\end{array}\right)$$

$$\xrightarrow{\frac{3}{7}R_3+R_1;\frac{12}{7}R_3+R_3}\left(\begin{array}{ccc|c}1&0&0&2\\0&1&0&1\\0&0&1&3\end{array}\right)\quad x=2, y=1, z=3 \quad \text{or } (2,1,3)$$

c.

$$\left(\begin{array}{cc|c}2&3&2\\1&3&-2\\1&-1&3\end{array}\right)\xrightarrow{R_1\leftrightarrow R_2}\left(\begin{array}{cc|c}1&3&-2\\2&3&2\\1&-1&3\end{array}\right)\xrightarrow{-2R_1+R_2;-1R_1+R_3}\left(\begin{array}{cc|c}1&3&-2\\0&-3&6\\0&-4&5\end{array}\right)\xrightarrow{R_2/-3}\left(\begin{array}{cc|c}1&3&-2\\0&1&-2\\0&-4&5\end{array}\right)$$

$$\xrightarrow{-3R_2+R_1;4R_2+R_3}\left(\begin{array}{cc|c}1&0&4\\0&1&-2\\0&0&3\end{array}\right)\quad \text{No Solution.}$$

d.

$$\left(\begin{array}{ccc|c}0&3&2&4\\2&-1&-3&3\\2&2&-1&7\end{array}\right)\xrightarrow{R_1\leftrightarrow R_2}\left(\begin{array}{ccc|c}2&-1&-3&3\\0&3&2&4\\2&2&-1&7\end{array}\right)\xrightarrow{R_1/2}\left(\begin{array}{ccc|c}1&-\frac{1}{2}&-\frac{3}{2}&\frac{3}{2}\\0&3&2&4\\2&2&-1&7\end{array}\right)\xrightarrow{-2R_1+R_3}\left(\begin{array}{ccc|c}1&-\frac{1}{2}&-\frac{3}{2}&\frac{3}{2}\\0&3&2&4\\0&3&2&4\end{array}\right)$$

$$\xrightarrow{R_2/3}\left(\begin{array}{ccc|c}1&-\frac{1}{2}&-\frac{3}{2}&\frac{3}{2}\\0&1&\frac{2}{3}&\frac{4}{3}\\0&3&2&4\end{array}\right)\xrightarrow{\frac{1}{2}R_2+R_1;-3R_2+R_3}\left(\begin{array}{ccc|c}1&0&-\frac{7}{6}&\frac{13}{6}\\0&1&\frac{2}{3}&\frac{4}{3}\\0&0&0&0\end{array}\right)$$

Infinitely many solutions.

$x-\frac{7}{6}x=\frac{13}{6}\Rightarrow x=\frac{7}{6}x+\frac{13}{6}$

$y+\frac{2}{3}z=\frac{4}{3}\Rightarrow y=-\frac{2}{3}z+\frac{4}{3}$

$\left(\frac{7}{6}x+\frac{13}{6},-\frac{2}{3}z+\frac{4}{3},z\right)$, z is any real number.

7. a. Infinitely many solutions given by:
x = -3
y – z = -17 => y = -17 + z

so, (-3, -17 + z, z) where z is any real number.

b. No solution.

c. Infinitely many solutions given by:
y + w = 3 => y = 3 – w
z – 2w = 4 => z = 4 + 2w

so, (x, 3 – w, 4 + 2w, w) where x and w are any real numbers.

d. x = 1, y = -8, z = 7 or (1, -8, 7)

8. a. u = 3, x = $\frac{5}{2}$, y = 7, z = 2
b. u = 1, x = 6, y = 2, z = 0
c. x = 16, y = $\frac{8}{3}$, z = $\frac{-4}{3}$
d. u = $\frac{10}{7}$, x = 12, y = $\frac{32}{3}$, z = $\frac{-16}{5}$

9. a. The matrix A is a 4x3. The matrix B is a 3x3. The matrix C is a 3x3.
b. $a_{31} = 15$, $b_{22} = 5$, $c_{23} = -4$
c. Matrices B and C.
d. $A^T = \begin{pmatrix} 2 & 1 & 15 & 10 \\ 7 & 0 & -4 & 7 \\ -8 & 11 & -11 & 6 \end{pmatrix}$; $B^T = \begin{pmatrix} 8 & -13 & 16 \\ -2 & 5 & 20 \\ 1 & 0 & 4 \end{pmatrix}$; $C^T = \begin{pmatrix} -8 & 21 & 9 \\ 10 & -7 & -10 \\ 17 & -4 & -14 \end{pmatrix}$
e. -11A + 3C is not possible.
f. 8B – 3C = $\begin{pmatrix} 88 & -46 & -43 \\ -167 & 61 & 12 \\ 101 & 190 & 74 \end{pmatrix}$

10. a. $\begin{pmatrix} -34 & -24 \\ 14 & -15 \\ -42 & -16 \end{pmatrix}$
b. $\begin{pmatrix} 45/2 & 37/2 \\ 123/2 & 145 \end{pmatrix}$

11. a. $\begin{pmatrix} -1 & 2 \\ -9 & -63 \\ 16 & 43 \end{pmatrix}$

b. $\begin{pmatrix} 8 & 20 & 15 \\ 6 & 8 & 20 \\ 8 & -32 & -7 \end{pmatrix}$

c. Not possible.

d. $\begin{pmatrix} 84 & 119 \\ 70 & 84 \\ -36 & -49 \\ 96 & 98 \end{pmatrix}$

e. $\begin{pmatrix} -47 & -118 & 4 & -39 & 25 \\ 65 & 18 & -2 & -55 & 23 \\ 15 & -78 & 26 & -57 & 73 \end{pmatrix}$

12. a. $\begin{pmatrix} 1 & -3 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} -2 & 3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b. $\begin{pmatrix} 2 & 4 & -2 \\ -4 & -6 & 1 \\ 3 & 5 & -1 \end{pmatrix}\begin{pmatrix} 1/2 & -3 & -4 \\ -1/2 & 2 & 3 \\ -1 & 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 1/2 & -3 & -4 \\ -1/2 & 2 & 3 \\ -1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 4 & -2 \\ -4 & -6 & 1 \\ 3 & 5 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

13. a. D = -6 – (-5) = -1; $A^{-1} = \begin{pmatrix} 3 & -5 \\ 1 & -2 \end{pmatrix}$

b. D = 50 – 2 = 48; $B^{-1} = \begin{pmatrix} 5/48 & -1/24 \\ -1/48 & 5/24 \end{pmatrix}$

c. D = -6 – (-6) = 0, so C does not have an inverse.

d. $D^{-1} = \begin{pmatrix} 1 & -1 & -1 \\ -6/5 & 7/5 & 4/5 \\ -2/5 & 4/5 & 3/5 \end{pmatrix}$

14. a. x = 1, y = 1

b. x = 15, y = -10

15. A maximum of 38 occurs at (2, 4).

16. A minimum of 70 occurs at (0, 2).

17. Let x = # bags of Best Food; y = # bags of Natural Nutri

The linear programming problem (LPP): Min $N = 8x + 3y$

$$\begin{aligned} \text{s. t. } 4x + 4y &\geq 1{,}000 \\ 2x + y &\leq 400 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

You should use 0 bags of Best Food and 250 bags of Natural Nutri. The amount of nitrogen that will be added is 750 pounds.

18. Let x = # of 2-person boats; y = # of 4-person boats

The linear programming problem (LPP): Max $P = 25x + 50y$

$$\begin{aligned} \text{s. t. } 0.9x + 1.2y &\leq 780 \\ 0.8x + 1.8y &\leq 950 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

They should produce 350 4-person boats and 400 2-person boats.