Math 1313
Finite Math

## Test 4 Supplemental Review

1. Let $A$ and $B$ be events in a sample space S . Suppose that $P(A)=.45, P(B)=.38$, and $P(A \cap B)=.21$. Find each of the following:
a. $P(A \mid B)$
b. $P(B \mid A)$
c. $P\left(A \mid B^{c}\right)$
d. $P\left(B \mid A^{c}\right)$
2. Suppose $A$ and $B$ are independent events and $P(A)=.25$ and $P(B)=.35$. Find each of the following:
a. $P(A \cap B)$
b. $P(A \cup B)$
3. A pair of fair dice is cast. Let E denote the event that the number landing uppermost on the first die is a 5 and let F denote the event that the sum of the numbers landing uppermost is 6 . Determine whether E and F are independent events.
4. Suppose that five green marbles and 8 yellow marbles are placed in an urn. An experiment consists of drawing 2 marbles from the urn in succession and without replacement. What is the probability that
a. both marbles drawn are the same color?
b. the second marble drawn is green?
c. the second marble drawn is green, given that the first marble drawn was yellow?
5. In a survey of 1000 eligible voters selected at random, it was found that $62 \%$ had a college degree. Additionally, the survey results showed that of the eligible voters who had a college degree, $85 \%$ voted in the most recent senatorial election. Furthermore, only $42 \%$ of the eligible voters without a college degree voted in the same election. Based on this information and assuming it is representative of the general voting population, find the probability that a randomly selected eligible voter
a. voted in the last senatorial election.
b. has a college degree and did not vote in the last senatorial election.
c. does not have a college degree and did not vote in the last senatorial election.
6. Use the same information as for problem 5 to answer these questions.
a. Find the probability that a randomly selected eligible voter has a college degree if it is known that s/he voted in the last senatorial election.
b. Find the probability that a randomly selected eligible voter does not have a college degree if it is known that s/he did not vote in the last senatorial election.
7. A national research institute published a study that showed that $45 \%$ of 10 -year-olds had never had a cavity, $39 \%$ of 11 -year-olds had never had a cavity, and $32 \%$ of 12 -year-olds had never had a cavity. If a child is selected at random from a group of fourteen 10-year-olds, eighteen 11-year-olds and nine 12-year olds, and this child does not have a cavity, what is the probability that the child is 12 years old?
8. Families in a certain community were surveyed. In this community $98 \%$ indicated that they own more than 2 vehicles. Of those that own more than 2 vehicles, $84 \%$ are a family of more than 2 . Of those that do not own more than 2 vehicles, $31 \%$ are not a family of more than 2 . If a family is chosen at random, what is the probability that that family
a. does not own more than 2 vehicles?
b. is a family of more than 2 , given that that family does not own more than 2 vehicles?
c. is a family of more than 2 ?
d. does not own more than 2 vehicles and is not a family of more than 2 ?
9. The results of a survey regarding position on a national lottery as an alternative to federal income tax are summarized in the table:

| Gender | Percentage of <br> Voters Surveyed | Percentage <br> Favoring Lottery | Percentage not <br> Favoring Lottery | Percentage with <br> No Opinion |
| :---: | :---: | :---: | :---: | :---: |
| Male | .56 | .74 | .22 | .04 |
| Female | .44 | .65 | .21 | .14 |

a. What is the probability that a registered voter who favored a national lottery was a woman?
b. What is the probability that a registered voter who expressed no opinion regarding the lottery was a man?
10. There are three urns, each of which contains some marbles. An experiment consists of selecting an urn and then selecting a marble from that urn. Suppose the urns are labeled X, Y and Z. Urn X contains 12 red marbles and 9 green marbles. Urn Y contains 15 red marbles and 12 green marbles. Urn Z contains 11 red marbles and 14 green marbles. Assume that the urns are equally likely to be chosen.
a. What is the probability that a green marble was chosen, given that Urn Y was chosen?
b. What is the probability that Urn Z was chosen, if we know that the marble selected was red?
c. What is the probability that a green marble was chosen?
11. A blue urn contains 4 green, 3 blue and 6 orange marbles. A green urn contains 6 green, 5 blue and 2 orange marbles. One of the two urns is chosen at random, with the blue urn being more likely to be chosen with probability 0.92 . Once an urn is chosen, then a marble is drawn at random from the chosen urn. What is the probability that
a. an orange marble is chosen, given that the green urn was chosen?
b. the blue urn and a green marble were chosen?
c. a blue marble is chosen?
d. the blue urn is chosen, given that the marble drawn was green?
12. A jar contains 10 red, 14 black and 12 yellow jellybeans. Two jellybeans are chosen at random, in succession and without replacement. What is the probability that
a. the second jellybean is black, given that the first one was yellow?
b. a red jellybean and then a yellow jellybean were chosen?
c. the two jellybeans are red?
d. the two jellybeans are the same color?
e. the first one was yellow, given that the second one was black?
13. Box \#1 contains 9 violet and 7 white rubber bands. Box \#2 contains 5 violet, 8 white and 6 blue rubber bands. Box \#3 contains 7 violet, 10 white and 4 blue rubber bands. The probabilities that Box \#1, Box \#2, and Box \#3 will be chosen are $3 / 8,2 / 5$, and $9 / 40$, respectively. A box is chosen and then a rubber band is chosen from the box. What is the probability that
a. a violet rubber band is chosen, given that Box \#3 was chosen?
b. a white rubber band is chosen?
c. a blue rubber bank is chosen?
d. Box \#2 is chosen, given that the rubber band drawn was violet?
e. Box \#3 is chosen, given that the rubber band drawn was blue?
14. In a certain community, $94 \%$ of the people in the age group 16-22 have a driver’s license. Records indicate that $85 \%$ of people in this age group that have a driver's license have had a minor car accident in the past 6 months, whereas $98 \%$ of people in this age group that do not have a driver's license have not had a minor car accident in the past 6 months. A person from this age group is chosen at random, what is the probability that
a. this person does not have a driver's license and has had a minor accident in the past 6 months?
b. has not had a minor accident in the past 6 months?
c. this person has a driver's license, given that he/she has not had a minor accident in the past 6 months?
15. A department store reports that $84 \%$ of its charge account holders are women, and that $98 \%$ of these accounts have always been paid on time, while $5 \%$ of the charge account holders that are men have not always been paid on time. An account is selected at random. What is the probability that that account
a. belongs to a woman and has not always been paid on time?
b. has always been paid on time?
c. has not always been paid on time, given that it belongs to a man?
16. A sociology graduate student conducted a survey in a certain town, 1,679 were men and 1,395 were women. She found that $67 \%$ of the men and $45 \%$ of the women smoke everyday. A person surveyed is chosen at random. What is the probability that the person
a. is male and smokes everyday?
b. does not smoke everyday, given that the person is male?
c. does not smoke everyday?
d. is female, given that this person smokes everyday?
17. A test for a certain disease was given to 2,000 people. Twelve percent were known to have this disease. For the people who had this disease, the test indicated the presence of the disease in $97 \%$ of the people, was inconclusive for $2.5 \%$, and no disease in $0.5 \%$. For the people who did not have the disease, the test indicated the presence of the disease in $0.5 \%$ of the people, was inconclusive for $0.5 \%$, and indicated no disease in $99 \%$. A person is chosen at random. What is the probability that
a. the test indicated that he/she has the disease, given the person does not have the disease?
b. the test indicates no disease?
c. the test was inconclusive?
d. the person has the disease, given that the test indicated that he/she has the disease?
18. All alarm clock radios produced at the ACLKR company must pass two inspections. Records indicate that the probability that the first inspector will miss a defective alarm clock radio is 0.04 . If a defective alarm radio clock gets past the first inspection, the probability that the second inspector will not detect it is 0.005 . What is the probability that a defective alarm clock will get past both inspectors?
19. Use the given probability distribution of the random variable $X$ to answer the questions which follow:

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(X=x)$ | .05 | .21 | .17 | .28 | .12 | .09 | .08 |

a. Find $P(X \geq 2)$
b. Find $P(X \leq 3)$
c. Find $P(0 \leq X \leq 3)$
d. Find $E(X)$
e. Find $\operatorname{Var}(X)$
f. Find $\sigma$
20. A group of people were surveyed about their reactions to a new soft drink. They were asked to rate the beverage on a scale of 1 to 10 , with 10 being the highest rating. The table that follows gives the results of the survey:

| Rating | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 14 | 21 | 25 | 45 | 41 | 67 | 62 | 74 | 31 | 20 |

Let the random variable X denote the rating given to the soft drink by a randomly chosen participant. Find the probability distribution of X and draw a histogram.
21. Find the expected value, variance and standard deviation of the random variable X in problem 20.
22. In a game of chance, 1000 tickets are sold for $\$ 5$ each. The prizes given are as follows: one grand prize of $\$ 2000$, one second place prize of $\$ 500$, one third place prize of $\$ 250$ and 10 consolation prizes of $\$ 25$ each. What are the expected net earnings of a person who buys one ticket?
23. The probability of an event E occurring is .85 . What are the odds in favor of E occurring?
24. Suppose you estimate the odds of landing a certain job as 5 to 2 . What is the probability that you will get the job?
25. Suppose your boss feels that the odds of getting a lucrative contract are 10 to 1 . What is the probability that he will not get the contract?
26. A probability distribution of a random variable has a mean of 42 and a standard deviation of
2. Use Chebychev's inequality to estimate the probability that an outcome of the experiment lies between 32 and 52 .
27. The average starting salary at a new graduate in a certain profession is $\$ 48,000$ with a standard deviation of $\$ 500$. Use Chebychev's inequality to estimate the probability that the starting salary of a new graduate in this profession will be between $\$ 46000$ and $\$ 50000$ ?
28. Consider the following binomial experiment. A heart transplant operation is considered a success if the patient survives one or more years after the surgery. Suppose the probability that a heart transplant operation is successful is .57 . Of ten patients who have recently undergone such an operation, what is the probability that one year from now
a. none of the heart recipients will be alive?
b. exactly three will be alive?
c. at least three will be alive?
d. all will be alive?
29. Find the mean, variance and standard deviation for the situation described in problem 28.
30. Consider the following binomial experiment. The probability that a randomly selected student at a certain college will graduate with a bachelor's degree after four years of study is estimated to be 0.87 . From among a group of 15 students at this college, what is the probability that
a. all of them will graduate after four years of study?
b. exactly 10 of them will graduate after four years of study?
c. at most 12 of them will graduate after four years of study?
d. at least one of them will graduate after four years of study?
31. Find the mean, variance and standard deviation for the situation described in problem 30.
32. A new drug has been tested and found to be effective in treating $83 \%$ of the people afflicted with a certain disease. If the drug is administered to 1250 people who have the disease, find the mean and the standard deviation of the number of people for whom the drug can be expected to be effective.
33. Suppose that $Z$ is the standard normal variable. Find each of the following probabilities:
a. $P(Z<2.55)$
b. $P(Z>-1.07)$
c. $P(-1.28<Z<1.89)$
d. $P(Z>0.78)$
e. $P(Z<-1.55)$
f. $P(0.5<Z<2.98)$
34. Suppose that Z is the standard normal variable. Find the value of z in each of the following:
a. $P(Z<z)=.7019$
b. $P(Z<-z)=.3228$
c. $P(Z>z)=.2296$
d. $P(Z>z)=.0162$
e. $P(-z<Z<z)=.9500$
f. $P(-z<Z<z)=.3544$
35. Suppose $X$ is a normal variable and $\mu=5$ and $\sigma=1.5$. Find each of the following probabilities:
a. $\mathrm{P}(X<2.2)$
b. $\mathrm{P}(X>3.42)$
c. $\mathrm{P}(-.08<X<6.22)$
36. Salespeople for a business machine company have average annual sales of $\$ 200,000$ with a standard deviation of $\$ 20,000$. What is the probability that a randomly selected salesperson will have annual sales of $\$ 240,000$ or more? Assume a normal distribution.
37. Scores on a nationwide standardized test are normally distributed. The scores are scaled so that the mean is 500 and the standard deviation is 100 . What is the probability that a randomly selected student who took the test scored
a. below 400 ?
b. between 400 and 600 ?
c. above 700 ?
d. between 450 and 650 ?
38. A marksman's chance of hitting a bulls-eye with each of his shots is $82 \%$. If he fires 30 shots, what is the probability of his hitting the target
a. at least 20 times?
b. fewer than 25 times?
c. between 21 and 26 times, inclusive?

