Math 1313 Section 3.4

Section 3.4: Matrix Multiplication

If A is a matrix of size m x n and B is a matrix of size n x p then the product AB is defined and is a matrix of size m x p.

So, two matrices can be multiplied if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

Example 1: Multiple the given matrices.

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is a 1 x 3 matrix $\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ is a 3 x 1 matrix

When multiplied the ending matrix will be 1 x 1.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

Here is how you multiply:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \begin{bmatrix} a_{11} \times b_{11} + a_{12} \times b_{21} \\ a_{21} \times b_{11} + a_{22} \times b_{21} \end{bmatrix}$$

Example 2: Multiply the given matrices.

a. $\begin{bmatrix} -2 & 4 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \\ 6 \end{bmatrix}$

Math 1313 Section 3.4

Example 3: Mike and Sam have stock as follows:

$$\mathbf{A} = \begin{array}{c} \text{BAC} \quad \text{GM} \quad \text{IBM} \quad \text{TRW} \\ \begin{bmatrix} 200 & 300 & 100 & 200 \\ 100 & 200 & 400 & 0 \end{bmatrix} \end{array}$$

Mike is this row one and Sam row two

At the close of trading on a certain day, the price $/share (GM, IBM, BAC, respectively) are:

$$\mathbf{B} = \begin{bmatrix} 54 \\ 48 \\ 98 \\ 82 \end{bmatrix}$$

AB =

Example 4: Multiply the following matrices if possible.

Let $A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 0 & 3 \\ 1 & 2 & -1 \end{pmatrix}$, $C = \begin{pmatrix} -10 & 9 \\ -6 & 4 \end{pmatrix}$, and $D = \begin{pmatrix} -3 & 9 \\ 6 & 1 \\ 0 & 9 \\ 8 & 4 \end{pmatrix}$ compute, if possible:

AB

CD

CA

Laws for Matrix Multiplication

If the products and sums are defined for the matrices A, B and C, then

1. (AB)C = A(BC)
2. A(B + C) = AB + AC

Note: In general, matrix multiplication is not commutative – that is, AB $\neq$ BA.

Example 5: If A and B are matrices we will look at the product AB and BA.

$$A = \begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 2 \\ 5 & 7 \end{bmatrix}$$

AB =

BA =

Math 1313 Section 3.4

Identity Matrix

The square matrix of size n having 1s along the main diagonal and zeros elsewhere is called the identity matrix of size n.

The **identity matrix** of size n is given by $I_n = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{pmatrix}$

If A is a square matrix of size n, then $I_n A = AI_n = A$.

Example 6: Given the following matrices,

$$X = \begin{pmatrix} 0 & 1 & -2 \\ 4 & -2 & 1 \\ 5 & 0 & -3 \end{pmatrix}, \qquad Y = \begin{pmatrix} 2 & 3 & -4 & 1 \\ -5 & 2 & 1 & 6 \\ 0 & -2 & 3 & -4 \end{pmatrix}$$

a. Is XY defined, if so what is the size?

b. Let A=XY, what is a_{23}?

Example 7: The following table displays the average grade in each category for an upper level honors course with 4 students.

	Test 1	Test 2	Test 3	Final Exam	Homework Avg	Quiz Avg
Mark	94	80	78	86	91	92
Ashley	80	88	90	85	76	100
Scott	100	75	88	82	84	88
Melissa	70	82	86	90	78	91

If each test is worth 16%, the final exam is worth 24%, the homework average is worth 12%, and the quiz average is worth 16%, what is each student's course average? Use a matrix to display the grades and another to display the percentages. Give the answer in the form of a matrix.