

**Section 5.4: Permutations and Combinations**

**Definition: n-Factorial**

For any natural number  $n$ ,  $n(n - 1)(n - 2) \dots 3 \cdot 2 \cdot 1$   
 $0! = 1$

A **permutation** is an arrangement of a specific set where the order in which the objects are arranged is important.

**Formula:**  $P(n, r) = \frac{n!}{(n-r)!}, \quad r \leq n$

where  $n$  is the number of distinct objects and  $r$  is the number of distinct objects taken  $r$  at a time.

**Formula: Permutations of  $n$  objects, not all distinct**

Given a set of  $n$  objects in which  $n_1$  objects are alike and of one kind,  $n_2$  objects are alike and of another kind, ..., and, finally,  $n_r$  objects are alike and of yet another kind so that

$$n_1 + n_2 + \dots + n_r = n$$

then the number of permutations of these  $n$  objects taken  $n$  at a time is given by

$$\frac{n!}{n_1! n_2! \dots n_r!}$$

A **combination** is an arrangement of a specific set where the order in which the objects are arranged is not important.

**Formula:**  $C(n, r) = \frac{n!}{r!(n-r)!}, \quad r \leq n$

where  $n$  is the number of distinct objects and  $r$  is the number of distinct objects taken  $r$  at a time.

**Example 1:** You are in charge of seating 5 honored guests at the head table of a conference. How many seating arrangements are possible if the 8 chairs are on one side of the head table?

**Example 2:** Find the number of ways 9 people can arrange themselves in a line for a group picture.

**Example 3:** An organization has 30 members. In how many ways can the positions of president, vice-president, secretary, treasurer, and historian be filled if not one person can fill more than one position?

**Example 4:** An organizations needs to make up a social committee. If the organization has 25 members, in how many ways can a 10 person committee be made?

**Example 5:** If there are 40 contestants in a beauty pageant, in how many ways can the judges award 1st prize and 2nd prize if not one person can be awarded 1st and 2nd?

**Example 6:** How many permutations can be formed from all the letters in the word MISSISSIPPI.

**Example 7:** A museum of fine arts owns 8 paintings by a given artist. Another fine arts museum wishes to borrow 3 of these paintings for a special show. How many ways can 3 paintings be selected for shipment out of the 8 available?

**Example 8:** A certain company has to transfer 4 of its 10 junior executives to a new location, how many ways can the 4 executives be chosen?

**Example 9:** A coin is tossed 5 times.

a. In how many outcomes do exactly 3 heads occur?

$\{(H_1H_2H_3TT), (H_1H_2T H_4T), (H_1H_2TT H_5), (H_1TH_3T H_5), (H_1TTH_4H_5), (H_1T H_3H_4T), (TH_2H_3H_4T), (TH_2H_3T H_5), (TH_2TH_4H_5), (TTH_3H_4H_5)\}$

b. In how many outcomes do at least 4 heads occur?

$\{(H_1H_2H_3H_4T), (H_1H_2H_3T H_5), (H_1H_2TH_4H_5), (H_1TH_3H_4H_5), (TH_2H_3H_4H_5)\}$

$\{(H_1H_2H_3H_4H_5)\}$



**Example 12:** A committee of 16 people, 7 women and 9 men, is forming a subcommittee that is to be made up of 6 women and 6 men. In how many ways can the subcommittee be formed?

**Example 13:** A computer store receives a shipment of 35 laser printers, including 6 that are defective. Five of these printers are selected to be displayed in the store.

a. How many of these selections will contain no defective printers?

b. How many of these selections will contain 1 defective printer?

c. How many of these selections will contain at least 1 defective printer?

**Example 14:** A customer at a fruit stand picks a sample of 6 avocados at random from a crate containing 35 avocados of which 8 are rotten. In how many ways can the batch contain at least 2 rotten avocados?