Math 1313 Section 6.6
Section 6.6 - Bayes' Theorem

In section 6.5, we were interested in finding the probability of the second event on a tree diagram if we know that the first event had occurred. In this section, we reverse the roles. If we know the second event has occurred, what is the probability of a given outcome of the first event?
$\mathrm{U}_{1}$ and $\mathrm{U}_{2}$ are mutually exclusive events. We would like to know: $\mathrm{P}\left(\mathrm{U}_{1} \mid \mathrm{E}\right)=$ Product of the branch probabilities leading to E through $\mathrm{U}_{1}$ / Sum of all branch products leading to E

$$
\mathbf{P}\left(\mathbf{U}_{\mathbf{1}} \mid \mathbf{E}\right)=\frac{\mathbf{P}\left(\mathbf{U}_{\mathbf{1}} \cap \mathbf{E}\right)}{\mathbf{P}\left(\mathbf{U}_{\mathbf{1}} \cap \mathbf{E}\right)+\mathbf{P}\left(\mathbf{U}_{\mathbf{2}} \cap \mathbf{E}\right)}
$$



Example 1: A company produces 1,000 refrigerators a week at three plants. Plant A produces 350 refrigerators a week, plant B produces 250 refrigerators a week, and plant C produces 400 refrigerators a week. Production records indicate that $5 \%$ of the refrigerators produced at plant A will be defective, $3 \%$ of those produced at plant B will be defective, and $7 \%$ of those produced at plant C will be defective. All the refrigerators are shipped to a central warehouse.
a. What is the probability that a refrigerator chosen at random from the warehouse will be defective?
b. If a refrigerator at the warehouse is found to be defective, what is the probability that it was produced at plant B?

Math 1313 Section 6.6
Example 2: In a random sample of 1,000 people it was found that $7 \%$ have a liver ailment. Of those who have a liver ailment, $40 \%$ are heavy drinkers, $50 \%$ are moderate drinkers and $10 \%$ are nondrinkers. Of those who do not have a liver ailment, $10 \%$ are heavy drinkers, $70 \%$ are moderate drinkers and $20 \%$ are nondrinkers. If a person is chosen at random and it is found that he or she is a heavy drinker, what is the probability of that person having a liver ailment?

Example 3: : Suppose that from a well-shuffled deck of 52 playing cards two cards are drawn in succession, without replacement. What is the probability that the first card was a king, given that the second card was not a king?

Math 1313 Section 6.6
Example 4: A placement test is given by a certain high school to predict student success in a particular math course. On average, $70 \%$ of students who take the test pass it, and $87 \%$ of those who pass the test also pass the course, whereas $8 \%$ of those who fail the test pass the course. If a student passed the course, what is the probability that he or she passed the test?

