Section 7.4: The Binomial Distribution

A binomial experiment has the following properties:

1. Number of trials is fixed.
2. There are 2 outcomes of the experiment. Success, probability denoted by p, and failure, probability denoted by q. Note $p + q = 1$
3. The probability of success in each trial is the same.
4. The trials are independent of each other.

Experiments with two outcomes are called **Bernoulli trials** or **Binomial trials**.

Finding the Probability of an Event of a Binomial Experiment:

In a binomial experiment in which the probability of success in any trial is *p*, the probability of exactly *x* successes in *n* independent trials is given by

$$P(X = x) = C(n, x)p^x q^{n-x}$$

X is called a **binomial random variable** and its probability distribution is called a **binomial probability distribution**. Example 1 in section 7.4 derives this formula.

Example 1: Consider the following binomial experiment. A fair die is cast four times. Compute the probability of obtaining exactly one 6 in the four throws.

Example 2: Let the random variable *X* denote the number of girls in a five-child family. If the probability of a female birth is 0.6, construct the binomial distribution associated with this experiment.

Example 3: Consider the following binomial experiment. If the probability that a marriage will end in divorce within 20 years after its start is 0.6, what is the probability that out of 6 couples just married, in the next 20 years

a. all will be divorced?

b. None will be divorced?

c. Exactly two couples will be divorced?

d. At least two couples will be divorced?

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Mean, Variance and Standard Deviation of a Random Variable

If *X* is a binomial random variable associated with a binomial experiment consisting of *n* trials with probability of success *p*, and probability of failure *q*, then the mean E(X), variance and standard deviation of *X* are given by applying the following formulas:

$\mu = E(X) = np$

$Var(X) = npq$

$\sigma = \sqrt{Var(X)} = \sqrt{npq}$

Example 5: The probability of a person contracting influenza on exposure is 62%. In the binomial experiment for a family of 12 that has been exposed, what is the:

a. mean?

b. standard deviation?

c. variance?

d. probability that at most 10 contract influenza?