Section 7.5: The Normal Distribution

A random variable that may take on infinitely many values is called a **continuous random variable**.

The probability distribution associated with this type of random variable is called a **continuous probability distribution.**

A continuous probability distribution is defined by a function f called the **probability density function**. The function has domain equal to those values the continuous random variable assumes.

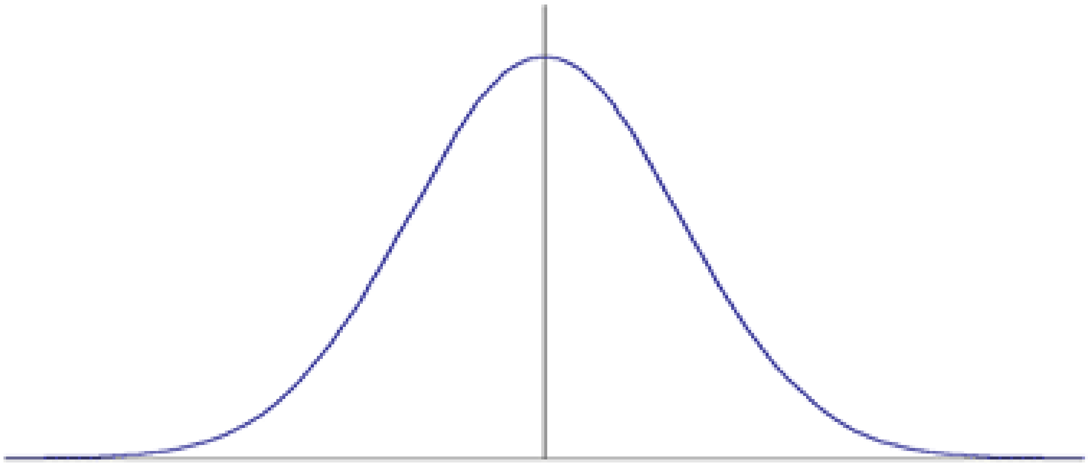
The **probability density function** has the following properties:

1. f(x) >0 for all values of x.

2. The area between the curve and the x axis is 1.

The probability that the random variable X associated with a given probability density function assumes a value in an interval $a < x < b$ is given by the **area of the region between the graph of *f* and the *x*-axis from *x* = *a* to *x* = *b*.**

Here is a picture:



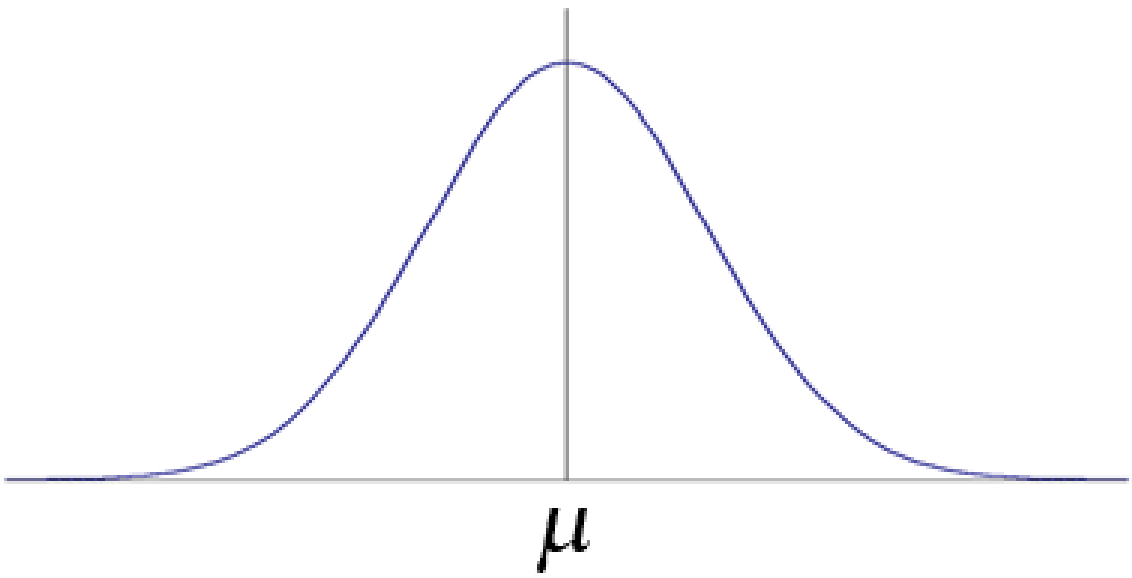
This value is $P(a < X < b)$

Note: $P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b) = P(a < X < b)$, since the area under one point is 0.

Normal Distributions.

For these types of distributions:
1. The graph is a bell-shaped curve.
2. μ and σ each have the same meaning (mean and standard deviation)
3. μ determines the location of the center of the curve.
4. σ determines the sharpness or flatness of the curve.

Also, the normal curve has the following characteristics:
1. The curve has peak at $x = \mu$.
2. The curve is symmetric with respect to the vertical line $x = \mu$.
3. The curve always lies above the *x*-axis but approaches the *x*-axis as *x* extends indefinitely in either direction.
4. The area under the curve is 1.
5. For any normal curve, 68.27% of the area under the curve lies within 1 standard deviation of the mean (i.e. between $\mu - \sigma$ and $\mu + \sigma$), 95.45% of the area lies within 2 standard deviations of the mean, and 99.73% of the area lies within 3 standard deviations of the mean.



Since any normal curve can be transformed into any other normal curve we will study, from here on, the **Standard Normal Curve**.

The **Standard Normal Curve** has μ =0 and σ =1.

The corresponding distribution and random variable are called **the Standard Normal Distribution** and the **Standard Normal Random Variable**, respectively.
The Standard Normal Variable will commonly be denoted *Z*.

The **area of the region under the standard normal curve to the left** of some value z, i.e. P$(Z < z)$ or P$(Z \leq z)$, is calculated for us in **Table I**.

Example 1: Let Z be the standard normal variable find the values of:

a. $P(Z < -1.91)$

b. $P(Z < 0.44)$

c. $P(Z > 0.50)$

d. $P(-1.65 < Z < 2.02)$

e. $P(1 < Z < 2.47)$

Example 2: Let Z be the standard normal variable. Find the value of z if z satisfies:

a. $P(Z < z) = 0.9495$

b. $P(Z > z) = 0.9115$

c. $P(Z < -z) = 0.6950$

d. $P(-z < Z < z) = 0.7888$

e. $P(-z < Z < z) = 0.8444$

When given a normal distribution in which $\mu \neq 0$ and $\sigma \neq 1$, we can transform the normal curve to the standard normal curve by doing whichever of the following applies.

$$P(X < b) = P\left(Z < \frac{b-\mu}{\sigma}\right)$$

$$P(X > a) = P\left(Z > \frac{a-\mu}{\sigma}\right)$$

$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

Example 3: Suppose X is a normal variable with $\mu = 80$ and $\sigma = 10$. Find:

a. $P(X < 100)$

b. $P(X > 65)$

c. $P(70 < X < 95)$