

# Math 3340 - Fixed Income Mathematics

## 1/24/2008 - Basics of Bonds

### Bond Markets

A *bond* is a contract between two parties, the purchaser and issuer. The *issuer* of the bond (usually a government, large financial institution, or bank) agrees to make payments (called coupon) at times between the date of purchase (called the *settlement date*) and the date of maturity of the bond. At maturity the issuer makes a final coupon payment and pays the face value (also called *nominal value* or *principal*) of the bond to the purchaser.

Note that a bond can have no coupon payments and consist of only a single payment of the face value at maturity. A bond that makes no coupon payments is called a *zero-coupon* bond. To an investor a zero-coupon bond derives its value from the fact that it is sold at a price below its face value. For this reason a zero-coupon bond is also called a *discount bond*.

Although the structuring of bond payments can be complex we will focus on standard bonds. A *standard bond* makes fixed coupon payments (assuming it is not a zero-coupon bond) periodically during its term.

Bonds are usually issued in order to raise capital. The U.S. government bonds raise capital in order to build roads for example. A school district or municipal authority issued a bond to build new classrooms or a civic center. Corporations issue bonds to finance expansion plans and investment strategies. Note that with any bond there is a risk of default, i.e. the issuer will not make good his promise to pay the face value. Default risk is an extremely difficult quantity to measure, partly because of the lack of market data. We will assume that the bonds are default free. For U.S. treasuries this is a reasonable assumption, since the issuer is the U.S. government.

The other kind of risk associated to bonds is the change in interest rates. We will return to this topic in a few weeks.

As a further simplification of matters we will deal with bonds issued by the United States Federal Reserve. There are three kinds of treasury securities issued by the government.

- **Treasury bill (T-bill)**. A treasury bill is a zero-coupon bond issued by the U.S. treasury. The term of these bonds is less than a year.
- **Treasury note (T-note)**. A T-note is a coupon bearing bond with a term between 2 and 10 years. The coupon payments are semi-annual.
- **Treasury bond (T-bond)**. A T-bond are the similar to T-notes, the only difference being that T-bonds have maturities between 10 and 30 years.

The Federal Reserve Bank of New York website <http://www.newyorkfed.org/aboutthefed/fedpoints.html> has some basic information about the mechanics of buying bonds and reading bond quotes. This is useful information for your own general knowledge, but you will not be tested over this!

When dealing with bonds (or any fixed income security) time is always a variable. when purchasing bonds it is important to know how much time is left to maturity, or how long it has been since the last coupon date. The convention used to count the time is called the day count. Unfortunately, there are several day count methods and each security has its own day count. For a T-bill the day count method is called actual/360. When doing calculations with a T-bill

we assume that a year has 360 days and in determining the time to maturity we count the actual number of days.

Here's what a treasury bill quote looks like in the Wall Street Journal from January 21st, 2008.

Maturity	Days to maturity	Bid	Asked	Chg	Asked yield
2008 Jul 17	177	2.77	2.76	-0.16	2.84

The first column is the date of maturity, in this case July 17th, 2008. The next column is the number of days to maturity. The third and fourth column are the asked and bid discount rates. Remember that a zero-coupon bond derives its value to an investor from the fact that it is sold below its face value. The discount yield is a percentage. If  $P$  is the price of the bond,  $F$  is the face value,  $t$  is the current day ( $t = 0$  is the date the bond is issued) and  $T$  is the day of maturity then the discount yield is given by

$$d = \left( \frac{F - P}{F} \right) \frac{360}{T - t} \quad (1)$$

If we rearrange this equation, then we see that

$$P = F \left( 1 - d \frac{T - t}{360} \right) \quad (2)$$

Equation (2) tells us that lower prices correspond to higher discount rates. You will notice that there are two discount rates, asked and bid. The *asked price* is the least amount for which an issuer is willing to sell a bond, the *bid price* is the highest price a purchaser is willing to pay for a bond. The asked price is always higher than the bid price and consequently the asked discount yield is lower than the bid discount yield. The bonds asked yield is another way of expressing the bonds price. The yield  $y$  is given by

$$y = \left( \frac{F - P}{P} \right) \frac{365}{T - t}. \quad (3)$$

If we rearrange this last expression we get

$$P(1 + y(T - t)365) = F \quad (4)$$

which shows us that the bond yield is the simple interest rate required to so that the future value of the price equals the face value. If we divide the equation in 3 by the expression in 1 we get

$$\frac{y}{d} = \frac{\left( \frac{F - P}{P} \right) \frac{365}{T - t}}{\left( \frac{F - P}{F} \right) \frac{360}{T - t}} \quad (5)$$

$$= \frac{365F}{360P} \quad (6)$$

and so  $y = \frac{365F}{360P}d$ .

In computing the discount yield we used a divisor of 360, while for the bond's investment yield we used 365. The investment yield is computed on an actual/365 basis.

Let's go back to the bond quote above. We see that the asked discount yield is 2.76% or 0.0276. Let us compute the price of a bond with a 1 million dollar face value. We have the relation

$$P = F\left(1 - \frac{d(T-t)}{360}\right) \quad (7)$$

$$= 1000000\left(1 - \frac{0.0276(177)}{360}\right) \quad (8)$$

$$= \$986,430 \quad (9)$$

The investment yield on this bond is

$$y = \frac{F - P}{P} \frac{365}{T - t} \quad (10)$$

$$= \frac{1000000 - 986,430}{986,430} \frac{365}{177} \quad (11)$$

$$= 0.02836 \equiv 2.84\% \quad (12)$$

which agrees with the Wall Street Journal Quote. Note the investment yield and bond discount yield are given to two decimal places. This means that any calculations we do with them will be subject to some rounding error. If you try to compute the price from the bond investment yield you will get

$$P = F\left(1 + \frac{y(T-t)}{365}\right)^{-1} \quad (13)$$

$$= 1000000\left(1 + 0.0284(177)(365)\right)^{-1} \quad (14)$$

$$= \$986,415 \quad (15)$$

Note that this differs from the price we calculated above. When computing the T-bill price we will always use the discount yield.

Treasury notes are bonds that have maturities between 2 and 10 years and Treasury bonds are bonds that are issued with maturities between 10 and 30 years. Both T-notes and T-bonds are coupon bearing bonds. Each of these bonds makes a coupon payment semi-annually.

Let us look at an sample quote from the Wall Street Journal January 23rd, 2008, for a T-note

Maturity	Coupon	Bid	Asked	Chg	Asked yield
2016 Nov 15n	4.625	109:16	109:17	+16	3.36

The n indicates that this is a note.

The maturity is the date on which the bond matures and makes a payment of the face value and the final coupon payment. The first coupon payment is made 6 months after the bond is issued. The coupon is given as a percentage of the bond's principal. There is, as with the T-bill, an asked and bid quote. This time however it is the price that has been quoted. The notation needs a little explaining. Bond quotes are made in increments of 1/32 of a dollar. The notation 109:16 means  $\$109 + \frac{16}{32} = \$109.50$ . The face value of all bonds in this quote is \$100 and so really the price is the percentage of the face value that we are paying for the bond. The price quoted is called the clean price. This is the price of the bond at the last coupon date, or at the issuance date if the bond

has not yet made a coupon payment. An investor who purchases this bond will pay the *dirty price* which is the sum of the *clean price* and the *accrued interest*.

Let us compute the accrued interest on the T-note. The accrued interest is calculated for T-notes and T-bonds on an actual/actual basis. The maturity of the bond is on 11/15/2016. This means that the bond makes semi-annual payments on the 15th of May and the 15th of November every year. The last coupon date (or the issuance date) is November 15th, 2007. (When we say today we mean the date these notes were written - January 24th, 2008!) Since that we've had 15 days in November, 31 days in December and 24 days in January. If we trade the bond today, then the settlement day will be tomorrow (it's always one working day after the trade date). Therefore when we make the payment for the bond a total of  $t = 15 + 31 + 25 = 71$  days would have elapsed since the last coupon (issuance) date. When I buy this bond I will pay it's clean price plus the amount of interest that has accrued since the last coupon date. The next coupon payment is on 5/15/2008 and the number of days between the two coupon payments is given below

Nov	15
Dec	31
Jan	31
Feb	29
Mar	31
Apr	30
May	15
Total	182

Hence, there are  $D = 182$  days in this coupon period. The coupon payment is 4.625 which is a semi-annual payment of  $c = \frac{4.625}{2} = 2.3125$ . The accrued interest  $AI$  is given by

$$AI = \frac{ct}{D} = 2.3125 \times \frac{71}{182} = 0.902129$$

The price paid for the bond would therefore be  $\$ 109 + (17/32) + 0.902129 = \$ 110.43$ . We will look at more examples in the online sessions.