POLYNOMIAL EXTENSIONS OPERATORS FOR H^1, $H(\mathrm{curl})$ and $H(\mathrm{div})$ SPACES ON A CUBE

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I will discuss the construction of polynomial extension operators for the polynomial spaces defined on the cube forming the exact sequence that make the following diagram commute.

$$\begin{array}{ccccccc}
W_p(\Omega) & \xrightarrow{\boldsymbol{\nabla}} & \boldsymbol{Q}_p(\Omega) & \underset{\boldsymbol{K}}{\overset{\mathrm{curl}}{\rightleftarrows}} & \boldsymbol{V}_p & \xrightarrow{\mathrm{div}} & Y_p(\Omega) \\
\gamma_0 \downarrow \uparrow \mathcal{L}_0^{(p)} & & \gamma_{\mathbf{t}} \downarrow \uparrow \mathcal{L}_{\mathbf{t}}^{(p)} & & \gamma_{\mathbf{n}} \downarrow \uparrow \mathcal{L}_{\mathbf{n}}^{(p)} & & \gamma_{avg} \downarrow \uparrow \mathcal{L}_{avg} \\
W_p(\partial\Omega) & \xrightarrow{\boldsymbol{\nabla}} & \boldsymbol{Q}_p(\partial\Omega) & \xrightarrow{\mathrm{curl}} & V_p(\partial\Omega) & \xrightarrow{\gamma_{avg}} & \mathbb{R}
\end{array} \tag{0.1}$$

The main result of the presented work[1] is the fact that the norms of the extension operators $\mathcal{L}_0^{(p)}, \mathcal{L}_{\boldsymbol{t}}^{(p)}, \mathcal{L}_{\boldsymbol{n}}^{(p)}$ and $\mathcal{L}_{avg}$ can be bounded uniformly in polynomial degree p. The result is crucial for the *Projection Based Interpolation* theory [3], convergence analysis of p and hp Finite Element Methods and hp-adaptivity [2, 4]. The proof is based on the classical method of separation of variables and its discrete counterparts. As a byproduct of the result, we propose a new way of teaching the separation of variables emphasizing the finite energy assumption.

References

[1] M. Costabel, M. Dauge, and L. Demkowicz. Polynomial extension operators for H^1, $\boldsymbol{H}(\mathbf{curl})$ and $\boldsymbol{H}(\mathrm{div})$ spaces on a cube. Technical report, IRMAR Rennes, 2007. accepted to *Math. Comp.*

[2] L. Demkowicz. *Computing with hp Finite Elements. I.One- and Two-Dimensional Elliptic and Maxwell Problems*. Chapman & Hall/CRC Press, Taylor and Francis, October 2006.

[3] L. Demkowicz. Polynomial exact sequences and projection-based interpolation with applications to Maxwell equations. In D. Boffi and L. Gastaldi, editors, *Mixed Finite Elements, Compatibility Conditions and Applications*, Lecture Notes in Mathematics. Springer-Verlag, 2007. see also ICES Report 06-12.

[4] L. Demkowicz, J. Kurtz, D. Pardo, M. Paszyński, W. Rachowicz, and A. Zdunek. *Computing with hp Finite Elements. II. Frontiers: Three-Dimensional Elliptic and Maxwell Problems with Applications*. Chapman & Hall/CRC, October 2007. in press.