Department of Mathematics

University of Houston

Scientific Computing Seminar

Prof. Roland Glowinski
Department of Mathematics
University of Houston

A Least-Squares/Fictitious Domain Method for Linear Elliptic Boundary Value Problems with Neumann or Robin Boundary Conditions: A Virtual Control Approach

Thursday, April 2, 2009
3 PM- 4 PM
Room 634 S&R1

Abstract: Motivated by the numerical simulation of particulate flow with slip-boundary conditions at the interface fluid-particles, we are going to address in this lecture the solution of the following elliptic problem

$$\alpha u - \mu \nabla^2 u = f \ in\ \Omega \setminus \bar{\omega}, \tag{1}$$

$$u = g_0 \ on\ \partial\Omega, \ \ \mu\left(\frac{\partial u}{\partial n} + \frac{u}{l}\right) = g_1 \ on\ \partial\omega, \tag{2}$$

by a fictitious domain method (new to the best of our knowledge); in (1), (2), Ω denotes a bounded domain of $\mathbf{R}^d$ and ω a sub-domain of Ω. Our approach relies essentially on the transformation of (1), (2) in a (virtual) control problem (in the sense of J.L. Lions) involving an extension of (1) (completed by $u = g_0$ on $\partial\Omega$) on the whole Ω, the restriction of the extended solution to $\Omega \setminus \bar{\omega}$ being the solution of (1), (2). From an algorithmic point of view, one solves the control problem by a least-squares conjugate gradient algorithm whose finite element implementation is rather easy, even if the mesh associated with Ω does not match the geometry of ω. Numerical experiments, including the generalization to the solution of parabolic equations with moving ω suggest optimal orders of convergence.