Department of Mathematics

University of Houston

Scientific Computing Seminar

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Discontinuous Petrov Galerkin Method (DPG) with Optimal Test Functions.

Thursday, March 21, 2013
3:00 PM- 4:00 PM
Room 646 PGH

Abstract:

I will give a short tutorial on the DPG method proposed by Jay Gopalakrishnan and myself four years ago, emphasizing the main points and illustrating them with numerical examples. Here is a few of them:

1. The DPG method is a minimum-residual method with the residual evaluated in a dual norm.
2. The method can be interpreted as a Petrov-Galerkin method with optimal test functions (realizing the sup in the inf-sup condition).
3. The optimal test functions are computed on the fly by inverting (approximately) the Riesz operator corresponding to the test space.
4. With broken test spaces and localizable norms, the inversion is done elementwise, i.e. the optimal test functions are computed within the element routine. This is more expensive then for standard FE method but it is compatible with the standard FE technology.
5. The main price paid for the localization is the presence of additional unknowns: traces and fluxes. Compared with standard conforming FE methods or hybridizable DG methods, the number of (non-local) unknowns doubles and it is of the same range as for DG methods. Contrary to DG methods based on numerical flux, in the DPG method, the flux enters as additional unknown.
6. The method can be interpreted as a preconditioned least squares method. The stiffness matrix is hermitian and positive-definite. Its condition number is the same as for standard FEs.
7. The formulation based on a first order system is very natural but not necessary. You can work with the second order equation if you wish. The key point is to break the test functions.
8. There is nothing exotic about the ultra-weak variational formulation behind the DPG method. If the operator is well posed in the L^2 sense (the operator is L^2 bounded below), the ultra-weak variational formulation is also well posed with the corresponding inf-sup constant being of the same order.
9. With the use of optimal test functions, the issues of approximability and stability are fully separated. This is illustrated by using hp-adaptivity.
10. The method is especially suited for singular perturbation problems e.g. convection-dominated diffusion, high wave number wave propagation, elasticity for thin-walled structures etc. For problems of this type, one can systematically design a test norm to accomplish robustness, i.e. a stability uniform in the perturbation parameter.
11. If you have a hybrid FE code, converting it to a DPG code is very easy.
12. The methodology extends to nonlinear problems. I will show examples for compressible NS equations.

This seminar is easily accessible to persons with disabilities. For more information or for assistance, please contact the Mathematics Department at 743-3500.