Department of Mathematics

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Scientific Computing Seminar

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High order divergence-conforming hybrid DG FEM for incompressible flows

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1:30 PM- 2:30 PM
Room 646 PGH

Abstract: In this talk we consider the discretization of the unsteady incompressible Navier-Stokes equations in a velocity-pressure formulation:

$$\left\{ \begin{array}{rll} \frac{\partial}{\partial t}u + \operatorname{div}(-\nu\nabla u + u \otimes u + pI) & = f & \text{in } \Omega \\ \operatorname{div} u & = 0 & \text{in } \Omega \end{array} \right. \tag{1}$$

with boundary conditions $u = u_D$ on $\Gamma_D \subset \partial\Omega$ and $(\nu\nabla u - pI) \cdot n = 0$ on $\Gamma_{out} = \partial\Omega \setminus \Gamma_D$. Here, ν is the constant kinematic viscosity, u the velocity, p the pressure, and f is an external body force. We present an efficient and high order accurate discretization method based on the following main ingredients:

First, we make a disctinction between stiff linear parts and less stiff non-linear parts with respect to their *temporal and spatial* treatment. We exploit this using *operator-splitting time integration schemes* which rely only on efficient solution strategies for two simpler sub-problems: a corresponding hyperbolic transport problem and an unsteady Stokes problem.

Secondly, for the hyperbolic transport problem a spatial discretization with an Upwind Discontinuous Galerkin (DG) method and an explicit treatment in the time integration scheme is rather natural and allows for an efficient implementation.

Thirdly, the discretization of the Stokes problems is tailored with respect to two important challenges: a proper treatment of the incompressibility constraint and efficient solution of arising linear systems. In order to fulfill the incompressibility constraint exactly we use an $H(\operatorname{div})$-conforming discretization of the velocity combined with discontinuous pressures. We discuss advantages of $H(\operatorname{div})$-conforming discretizations such as energy-stability, pressure-robustness and Re-semi-robust error bounds. To enforce continuity of the velocity (weakly) also in tangential direction we apply a *Hybrid* DG formulation. For the task of solving linear systems, a discretization with Hybrid DG methods is better suited than standard DG methods. To improve the efficiency even further we apply a projection operator in the Hybrid DG formulation which allows to reduce the unknowns on element interfaces and thereby the number of globally coupled unknowns. To reduce the computational costs we introduce a modified version of the discretization using a *relaxed* $H(\operatorname{div})$-conforming finite element space.

We present the method(s), discuss aspects of the temporal and spatial discretization, implementational aspects and numerical results.