

Department of Mathematics

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Scientific Computing Seminar

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Stability of Low-Rank Tensor Representations and Structured Multilevel Preconditioning for Elliptic PDEs

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1:30 PM- 2:30 PM

Room 646 PGH

Abstract:

Folding grid-value vectors into high-dimensional tensors of mode sizes $2 \times 2 \times \cdots \times 2$, combined with low-rank representation in the tensor-train format, affords highly efficient approximations for various classes of functions. These include solutions of elliptic PDEs on nonsmooth domains or with oscillatory data. This tensor-structured approach is attractive because it leads to highly compressed, adaptive approximations based on simple discretizations. Straightforward choices of the underlying bases lead to the well-known *matrix ill-conditioning* of discrete operators.

In this work, we demonstrate that the use of tensor structure leads to *representation ill-conditioning*, a new effect specific to computations in tensor networks. We construct an explicit tensor-structured representation of a BPX preconditioner with ranks independent of the number L of discretization levels. However, the application of the preconditioner in this representation introduces representation ill-conditioning. To remedy this, we obtain a reduced-rank decomposition, which turns out to be free of both matrix and representation ill-conditioning.

For an iterative solver based on the soft thresholding of low-rank tensors, we obtain convergence and complexity estimates and demonstrate its reliability and efficiency for discretizations with up to 2^{50} nodes in each dimension.

- This is a joint work with Markus Bachmayr (University of Bonn).