UNIVERSITY of HOUSTON

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Scientific Computing Seminar

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The moving surface Navier-Stokes equations: Derivation and discretization

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1 PM- 2 PM
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Abstract: The motion of an inextensible viscous fluid layer represented by a material surface can be described by the Navier-Stokes equations. These equations are well understood in the bulk case, where the fluid occupies a three-dimensional area in $\mathbb{R}^3$. However, in the last decade, the interest in the Navier-Stokes equations posed on an evolving surface has grown. We derive the equations by using surface conservation laws and compare the derivation with different physical and mathematical approaches to derive evolving surface Navier-Stokes equations. The next step towards solving these equations is solving the tangential part of the system. Therefore, we introduce a method for the numerical solution of a simplified problem consisting of tangential surface Navier-Stokes equations (TSNSE) posed on a passively evolving smooth closed surface embedded in $\mathbb{R}^3$. For discretization of the TSNSE, we consider a geometrically unfitted finite element method known as TraceFEM. The TraceFEM applies to a fully Eulerian formulation of the problem and does not require a surface triangulation, which makes it convenient for deforming surfaces. In TraceFEM, one uses standard (bulk) finite element spaces to approximate unknown quantities on the surface $\Gamma(t)$ which propagates through a given triangulation of an ambient volume Ω, i.e. $\Gamma(t) \subset \Omega$ for all times t. The discrete formulation does not need a surface parametrization and uses tangential calculus in the embedding space $\mathbb{R}^3$. For scalar PDEs on evolving surfaces, variants of TraceFEM are known in the literature. For the TSNSE we choose a hybrid (finite difference in time - finite elements in space) approach since it is more flexible in terms of implementation and the choice of elements. We use a BDF scheme for the time-discretization and a stable Taylor-Hood pair of finite elements in space. To represent the surface, we use a level-set description and a higher-order method to calculate integrals on the surface approximation. Theoretical results show the optimal order of convergence. In this presentation, we explain the method and present numerical experiments that illustrate the optimality of the convergence.