The Cauchy-Euler Equation

The differential equation

(1)

or its equivalent

(1)

where each of a and b is a number is known as the Cauchy-Euler differential equation. Its is the function Q given by

for all complex numbers r.

Theorem. (1) If then y is a solution to (1) if and only if

for all $x > 0$ and some pair of numbers (c_1, c_2).

(2) If , then y is a solution to (1) if and only if

for all $x > 0$ and some pair of numbers (c_1, c_2).

(3) If where each of α and β is real and $\beta \neq 0$,then y is a solution to (1) if and only if

for all $x > 0$ and some pair of numbers (c_1, c_2).

Suggestion for Proof. Let u and y be related by

for all x

or

for $\dot{x} > 0$

and let

or .

Show that y satisfies (1) if and only if u satisfies the constant coefficient equation

for all t. (2)