EMCF Quiz 4 Due Wednesday, February 5 at 11:59 PM

1. Given the linear operator $L[y] = y' + \dfrac{1}{x}y$. Calculate $L(2x^2 + 3x)$.

   (a) $2x + 3$
   (b) $6x + 6$
   (c) $4x - 6$
   (d) $6x + 8$
   (e) None of the above.

2. $L[y] = y'' - \dfrac{4}{x}y' + \dfrac{6}{x^2}y$ is a linear operator. Calculate $L(2x^3 + 3x)$.

   (a) $4x - 3/x$
   (b) $2x + 6/x$
   (c) $x^2 - 2/x$
   (d) $6/x$
   (e) None of the above.

3. The family of orthogonal trajectories of the family $y^2 = Cx^3 - 2$ is:

   (a) $2x^2 - 3y^2 - 6\ln y = C$
   (b) $3y^2 - 2x^2 + 6\ln y = C$
   (c) $2x^2 + 3y^2 + 12\ln y = C$
   (d) $2x^2 + 3y^2 - \dfrac{12}{y^2} = C$
   (e) None of the above.

4. The family of orthogonal trajectories for the family of parabolas with axis parallel to the y-axis and vertex at the point $(-2, 4)$ is:

   (a) $(x + 2)^2 + 2(y - 4)^2 = C$
   (b) $y - 4 = \dfrac{C}{x + 2}$
   (c) $(x - 2)^2 - 2(y - 4)^2 = C$
   (d) $x + 2 = C(y - 4)^2$
   (e) None of the above.

5. If $1000 is deposited in a bank that pays 4.5% interest compounded continuously, then the amount in the account at the end of 10 years is: (Hint: The population growth law applies.)

   (a) $1491.82
   (b) $1568.31
   (c) $1603.42
   (d) $1648.72
   (e) None of the above.

6. A laboratory has 75 grams of a certain radioactive material. Two years ago, it had 100 grams. How much will the laboratory have 4 years from now?

   (a) 39.57 grams

   (b) 52.19 grams

   (c) 42.19 grams

   (d) 57.24 grams

   (e) None of the above.

7. Scientists observed that a small colony of penguins on a remote Antarctic island obeys the population growth law. There were 2000 penguins initially and 3000 penguins 12 months later. How long will it take for the number of penguins to double?

   (a) 1.35 years

   (b) 2.56 years

   (c) 2.12 years

   (d) 1.71 years

   (e) None of the above.

8. What is the half-life of a radioactive material if it takes 6 months for 1/4 of the material to decay?

   (a) 12.76 months

   (b) 15.55 months

   (c) 16.84 months

   (d) 14.46 months

   (e) None of the above.

9. At 12 noon on Jan. 1, the count in a bacteria culture was 400; at 4:00 pm the count was 1200. Let $P(t)$ denote the bacteria count at time t and assume that the culture obeys the population growth law. What was the bacteria count at 9 am on Jan. 1?

   (a) 231

   (b) 198

   (c) 175

   (d) 214

   (e) None of the above.

10. A thermometer is taken from a room where the temperature is $72^o F$ to the outside where the temperature is $32^o F$. After 2 minutes, the thermometer reads $48^o F$. How many minutes does the thermometer have to be outside for it to read $36^o F$?

   (a) 5.02 min

   (b) 6.29 min

   (c) 5.62 min

   (d) 4.73 min

   (e) None of the above.

11. A 100-gallon barrel, initially half-full of oil, develops a leak at the bottom. Let $A(t)$ be the amount of oil in the barrel at time t. Suppose that the amount of oil is decreasing at a rate proportional to the product of the time elapsed and the amount of oil present in the barrel. The mathematical model is

(a) $\dfrac{dA}{dt} = kA, \quad A(0) = 0$

(b) $\dfrac{dA}{dt} = ktA, \quad A(0) = 50$

(c) $\dfrac{dA}{dt} = tA, \quad A(0) = 100$

(d) $\dfrac{dA}{dt} = k(t + A), \quad A(0) = 50$

(e) None of the above.

12. Using the information in Problem 11, suppose that 20 gallons of oil leak out in the first 2 hours. Then, the amount of oil in the barrel at time t is

(a) $A(t) = 50 \left(\dfrac{3}{5}\right)^{t^2/4}$

(b) $A(t) = 50 \left(\dfrac{3}{5}\right)^{t/4}$

(c) $A(t) = 50 \left(\dfrac{2}{5}\right)^{t^2/2}$

(d) $A(t) = 50 \left(\dfrac{2}{5}\right)^{t^2/4}$

(e) None of the above.

13. A disease is spreading through a troop of 100 Monkeys. Let M(t) be the number of sick monkeys t days after the outbreak. The disease is spreading at a rate proportional to the number of monkeys who do not have the disease. Suppose that 10 monkeys had the disease initially. The mathematical model is:

(a) $\dfrac{dM}{dt} = ktM, \quad M(0) = 10$

(b) $\dfrac{dM}{dt} = kM(100 - M), \quad M(0) = 10$

(c) $\dfrac{dM}{dt} = k(100 - M), \quad M(0) = 10$

(d) $\dfrac{dM}{dt} = kt(100 - M), \quad M(0) = 10$

14. Using the information in Problem 13, suppose that 40 monkeys have the disease after 6 days. Then the number of sick monkeys (rounded off to the nearest monkey) after 18 days is

(a) 81

(b) 68

(c) 62

(d) 73

(e) None of the above.