EMCF Quiz 5 Due Tuesday, February 11 at 11:59 PM

1. The differential equation that has $y = C_1 x^3 + C_2$ as its general solution is:

   (a) $y'' - 2y = 0$.
   (b) $xy'' - 2y' = 0$.
   (c) $x^2 y'' - 2y = 0$.
   (d) $y'' - 2xy' = 0$.
   (e) none of the above.

2. The general solution of $xy' + 3y = \dfrac{e^x}{x}$ is:

   (a) $y = \dfrac{e^x - xe^x}{x^2} + \dfrac{C}{x^2}$.
   (b) $y = x^2 e^x - x^3 e^x + Cx^3$.
   (c) $y = \dfrac{xe^x - e^x}{x^2} + \dfrac{C}{x}$.
   (d) $y = \dfrac{xe^x - e^x}{x^3} + \dfrac{C}{x^3}$.
   (e) none of the above.

3. If $y = y(x)$ is the solution of the initial-value problem

$$xy' + y = 3\cos x, \quad y(\pi) = 0,$$

   then $\lim\limits_{x \to 0} y(x)$ is:

   (a) -3.
   (b) 3.
   (c) $3/\pi$.
   (d) does not exist.
   (e) none of the above.

4. The general solution of $yy' = xy^2 - x - y^2 + 1$ is:

   (a) $y^2 = Ce^{x^2 - 2x} + 1$
   (b) $y^2 - 1 = e^{x^2 - 2x} + C$
   (c) $y^2 = e^{x^2 - 2x} + C$
   (d) $y^2 - 1 = Ce^{2x - x^2}$
   (e) none of the above.

5. The general solution of $y' + xy = xy^3$ is

(a) $y = \dfrac{1}{1 + Ce^{x^2}}$

(b) $y^2 = \dfrac{1}{1 + Ce^{x^2}}$

(c) $y = \sqrt{1 + Ce^{x^2}}$

(d) $y^2 = \dfrac{1}{1 + Ce^{-x^2}}$

(e) none of the above.

6. The general solution of $y' = \dfrac{x^2 e^{y/x} + y^2}{xy}$. is

(a) $ye^{y/x} + xe^{y/x} = Cx - x \ln x$

(b) $ye^{-y/x} + xe^{-y/x} = x - x \ln x + C$.

(c) $ye^{-y/x} + xe^{-y/x} = Cx - x \ln x$

(d) $ye^{-y/x} + xe^{-y/x} = Cx + x \ln x$

(e) none of the above.

7. The family of orthogonal trajectories of $y^3 = Ce^{2x} + 2$ is:

(a) $3x + y^2 + \dfrac{4}{y} = C$

(b) $y^2 + 4 \ln y = 3x + C$

(c) $3x + y^2 + \dfrac{4}{y^3} = C$

(d) $y^2 + \dfrac{4}{y} = 3x + C$

(e) none of the above.

8. A sample of 100 grams of radioactive material was present initially and after 3 hours the sample lost 20% of its mass. An expression for the mass of the material remaining at any time t is:

(a) $A(t) = 100 \left(\dfrac{4}{5}\right)^{-t/3}$

(b) $A(t) = 100 \left(\dfrac{4}{5}\right)^{t/3}$

(c) $A(t) = 100 \left(\dfrac{1}{5}\right)^{t/3}$

(d) $A(t) = 100 \left(\dfrac{1}{5}\right)^{-t/3}$

(e) None of the above.

9. A biologist observes that a certain bacterial colony triples every 4 hours and after 12 hours occupies 1 square centimeter. Assume that the colony obeys the population growth law. The area the colony occupied when first observed was:

   (a) $\dfrac{1}{9}$ sq. cm.

   (b) $\dfrac{1}{81}$ sq. cm.

   (c) $\dfrac{1}{36}$ sq. cm.

   (d) $\dfrac{1}{27}$ sq. cm.

   (e) None of the above.

10. A disease is spreading through a small cruise ship with 200 passengers. Let $P(t)$ be the number of people who have disease at time t. The disease is spreading at a rate proportional to the product of the time elapsed and the number of people who are not sick. Suppose that 20 people have the disease initially. The mathematical model for the spread of the disease is:

   (a) $\dfrac{dP}{dt} = k(200 - P),\ P(0) = 20.$

   (b) $\dfrac{dP}{dt} = kt(200 - P),\ P(0) = 20.$

   (c) $\dfrac{dP}{dt} = ktP,\ P(0) = 20.$

   (d) $\dfrac{dP}{dt} = kP(200 - P),\ P(0) = 20.$

   (e) None of the above.

11. Refer to Problem 10. Suppose that 50 people are sick after 4 days. Then the number of people that are sick at any time t is given by:

   (a) $P(t) = 200 - 180 \left(\dfrac{5}{6}\right)^{t/4}$

   (b) $P(t) = 20 \left(\dfrac{5}{2}\right)^{t^2/16}$

   (c) $P(t) = 200 - 180 \left(\dfrac{5}{6}\right)^{t^2/16}$

   (d) $P(t) = \dfrac{200}{2 + 18(5/6)^{t/4}}$

   (e) None of the above.

12. $y'' - \dfrac{2}{x} y' - \dfrac{10}{x^2} y = 0$ has solutions of the form $y = x^r$. The general solution of the equation is:

(a) $y = C_1 x^2 + C_2 x^{-5}$

(b) $y = C_1 x^9 + C_2 x^{-1}$

(c) $y = C_1 x^{-2} + C_2 x^5$

(d) $y = C_1 x^2 + C_2 x^5$

(e) None of the above.

13. Find the solution of the initial-value problem

$$x^2 \, y'' - 6 \, y = 0, \quad y(1) = 6, \ y'(1) = -2.$$

Hint: The equation has solutions of the form $y = x^r$.

(a) $y = 2x^3 + 4x^{-2}$

(b) $y = 4x^6 - 2x^{-1}$

(c) $y = 4x^3 + 2x^{-2}$

(d) $y = 2x^{-6} + 4x$

(e) None of the above.

14. The general solution of $y'' - 8y' + 20y = 0$ is:

(a) $y = C_1 e^{2x} \cos 4x + C_2 e^{2x} \sin 4x$

(b) $y = C_1 e^{4x} \cos 2x + C_2 e^{4x} \sin 2x$

(c) $y = C_1 e^{10x} + C_2 e^{-2x}$

(d) $y = C_1 e^{5x} + C_2 e^{4x}$

(e) None of the above.

15. A fundamental set of solutions of $y'' - 4y' - 12y = 0$ is:

(a) $\{e^{4x}, \ e^{-3x}\}$

(b) $\{e^{6x}, \ e^{-2x}\}$

(c) $\{e^{2x}, \ e^{-6x}\}$

(d) $\{e^{-4x}, \ e^{3x}\}$

(e) None of the above.

16. The general solution of $y'' + 10y' + 25y = 0$ is:

   (a) $y = C_1 e^{5x} + C_2 x e^{5x}$
   (b) $y = C_1 e^{-5x} + C_2 e^{5x}$
   (c) $y = C_1 e^{5x} + C_2 x e^{-5x}$
   (d) $y = C_1 e^{-5x} + C_2 x e^{-5x}$
   (e) None of the above.

17. A solution basis for $y'' + 8y' + 16y = 0$ is:

   (a) $\{e^{4x}, xe^{4x}\}$
   (b) $\{e^{4x}, e^{-4x}\}$
   (c) $\{e^{-4x}, xe^{-4x}\}$
   (d) $\{e^{4x}, xe^{-4x}\}$
   (e) None of the above.

18. The second order linear differential equation that has $y = 2e^{-2x} - e^{4x}$ as a solution is:

   (a) $y'' - 2y' - 8y = 0$
   (b) $y'' - 6y' + 8y = 0$
   (c) $y'' + 2y' - 8y = 0$
   (d) $y'' - 6y' - 8y = 0$
   (e) None of the above.

19. The second order linear differential equation that has $y = 4xe^{-3x}$ as a solution is:

   (a) $y'' - 6y' + 9y = 0$
   (b) $y'' + 3y' = 0$
   (c) $y'' + 6y' + 9y = 0$
   (d) $y - 9y = 0$
   (e) None of the above.

20. The second order linear differential equation that has $y = 3e^{2x} \sin 2x$ as a solution is:

   (a) $y'' + 4y' + 12y = 0$
   (b) $y'' - 4y' + 8y = 0$
   (c) $y'' - 8y' + 8y = 0$
   (d) $y'' + 4y' + 8y = 0$
   (e) None of the above.