Section 1.2

Parameterized Families

We will introduce the notion of a parameterized family of function by way of some examples.

Example 1. Suppose that $\mathcal{F} = \{f|f(x) = 3x + C$ for all x and some number $C\}$. This is an example of a one-parameter family of functions. [ ]

Example 2. Suppose that

$$\mathcal{F} = \{f|f(x) = C_1x^2 + C_2 \text{ for all } x \text{ and some pair of number } C_1 \text{ and } C_2\}.$$

This set of quadratic functions is an example of a two-parameter family of functions. [ ]

General Solutions, **Singular Solutions**, **and Particular Solutions**

The set of solutions to an n-th order differential equation is often contained in an n-parameter family of functions. An expression that indicates this family is sometimes called the **general solution** of the differential equation.

Example 3. We will learn later in this course that y is a solution to

$$y'' + 4y = 0 \tag{1}$$

on the set of real numbers if and only if

$$y = C_1() + C_2() \tag{2}$$

for some pair of numbers C_1 and C_2 and all real numbers x. [ ]

In some cases, most solutions to a differential equation will be containedn in members of a parameterized family. Those solutions, if any, that are not members of the family are known as **singular solutions**.

Example 4. Most solutions of

$$y' = y^2$$

are contained in members of the one-parameter family indicated by

$$y = ().$$

However, the function given by

$$y = ()$$

is also a solution. [ ].

If specific values are assigned to the parameters in a parameterized family of solutions, the result is known as a **particular solution**.

Example 5. In view of Example 3, the function y given by

$$y = ()$$

is a particular solution to

$$y'' + 4y = 0.$$

Finding the Differential Equation

Sometimes a differential equation can be reconstructed from a parameterized family of solutions. If the family contains n parameters, we look for an n-th order differential equation.
[ ]

Example 6. Find a differential equation for the family indicated by

$$y = Cx^3 + 1$$

Solution. From

$$y = Cx^3 + 1 \tag{3}$$

we have

$$y' = (). \tag{4}$$

From (4) we have

$$C = ().$$

Using this expression for C, equation (3) becomes

$$y = () + 1 \text{ so } y = () + 1.$$

Thus

$$() \text{ so } ().$$

Example 7. Find a differential equation for the family indicated by

$$y = C_1 x + \frac{C_2}{x}.$$

Solution. From

$$y = C_1 x + \frac{C_2}{x} = C_1 x + C_2 x^{-1} \tag{5}$$

we have

$$y' = () \tag{6}$$

and

$$y'' = ().$$ 7

From (7) it follows that

$$C_2 = ().$$ 8

Then from (6) it follows that

$$C_1 = y' + C_2x^{-2} = y' + ()x^{-2} \text{ so } C_1 = y' + ()$$ 9

Putting the expressions for C_1 and C_2 given in (8) and (9) into (5), we have

$$y = C_1x + C_2x^{-1} = []x + []x^{-1}.$$

so

$$y = xy' + \frac{1}{2}x^2y'' + \frac{1}{2}x^2y''.$$

This simplifies to

$$x^2y'' + xy' - y = 0.$$

Additional Examples: See Section 1.2 of the text.

Suggested Problems. Do the odd numbered problems 11- 25 at the end of Section 1.3. Answers are posted on Dr. Walker's web site. There are no problems at the end of Section 1.2.