Section 1.3

Initial Conditions.

A list of the form

$$y(x_0) = k_0, y'(x_0) = k_1, y''(x_0) = k_2, \ldots, y^{(n-1)}(x_0) = k_{n-1}$$

is called a set or list of n initial conditions. Such a list of conditions together with an n-th order differential equation is called an n-th order **initial value problem**. It is desirable that initial value problems have unique solutions on some interval.

If the solutions to a differential equation are contained in an n-parameter family, then a list of n initial conditions will often determine the value of each parameter in the family.

Example. It is true that

$$y'' - 4y = 0 \qquad 1$$

if and only if

$$y = () \qquad 2$$

for some pair of numbers C_1 and C_2.

Suppose that y is a solution and y also satisfies the initial conditions

$$y(0) = 1 \text{ and } y'(0) = -1. \qquad 3$$

From

$$y = () \qquad 2$$

and $y(0) = 1$ we have, setting $x = 0$ and $y = 1$, that

$$1 = ()$$

or

$$() = 1. \qquad 4$$

Also from (2) we have

$$y' = ();$$

so using $y'(0) = -1$ we have, setting $x = 0$ and $y' = -1$ that,

$$-1 = ()$$

or

$$() = -1. \qquad 5$$

$$() = 1. \quad 4$$

$$() = -1. \quad 5$$

Solving (4) and (5) [ ] we have

$$C_1 = () \text{ and } C_2 = ().$$

Thus the solution to the initial value problem consisting of (1) and (3) is the function y given by

$$y = ().$$

Example. Suppose that an object moves along a coordinatized straight line and its displacement from the origin at time t is $y(t)$. Then its velocity $v(t)$ at time t is given by

$$v(t) = y'(t)$$

and its acceleration $a(t)$ is given by

$$a(t) = v'(t)$$

so

$$y''(t) = a(t).$$

Now suppose that a is constant with

$$a(t) = a$$

and that these initial conditions are satisfied:

$$y(0) = y_0 \text{ and } v(0) = y'(0) = v_0$$

Then

$$v'(t) = a.$$

Since

$$\int a dt = ()$$

we have

$$v(t) = () + C_1$$

and since $v(0) = v_0$ we have () yielding $C_1 = ()$. Thus

$$v(t) = ().$$

Continuing with

$$v(t) = ()$$

we have

$$y'(t) = ().$$

Since

$$\int () dt = ()$$

we have

$$y(t) = () + C_2.$$

Since $y(0) = y_0$ we have $()$ yielding $C_2 = ()$. Thus

$$y(t) = ().$$

Example. Find a differential equation for the family indicated by

$$y^2 = Cx^4 - 2. \qquad 1$$

Solution. Differentiating each side of (1) we have

$$() \text{ so } C = ().$$

From (1)

$$y^2 = ()x^4 - 2 \text{ or } 2y^2 = ()$$

so

$$y' = \frac{}{}$$

Example. Find a differential equation for the family indicated by

$$y = Ce^x + \sin x. \qquad 1$$

Solution. From (1) we have

$$y' = (). \qquad 2$$

Subtracting (2) from (1) we have

$$y - y' = ()$$

or

$$y' - y = ().$$

Example. Find a differential equation for the family indicated by

$$y = C_1 e^x + C_2 e^{-2x}. \qquad 1$$

Solution.

$$y' = () \qquad 2$$

$$y'' = (). \qquad 3$$

Subtracting (2) from (1) yields

$$y - y' = ().$$

Subtracting (3) from (1) yields

$$y - y'' = () \text{ so } y - y'' = -(y - y').$$

Thus

$$y'' + y' - 2y = 0$$

Additional Examples: See Section 1.3 of the text.

Suggested Problems: Do the odd numbered problems 1-25 for Section 1.3 The answers are posted on Dr. Walker's web site.