Section 2.3

Section 2.3 Two More First Order Differential Equations

Definition. A differential equation that is equivalent to one of the form

$$y' + p(x)y = q(x)y^r$$

where $r \neq 0$ and $r \neq 1$ is said to be a **Bernoulli** differential equation. When the equation is in this form, it can be solved

The result is a first order linear differential equation for v.

find y.

Example. Find all solutions to

 $y' + \frac{1}{x}y = 3x^2y^2$ on intervals of positive numbers.

Solution. Multiply each side by to get

Let

so

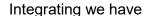
or

The integrating factor for this first order linear d.e. is

so

or

$$\left(\frac{1}{x}v\right)' = -3x.$$



so

Since

it follows that

or

The ${\it C}$ in the last equation is twice the ${\it C}$ in the previous equation. is a singular solution.

Saying that a differential equation of the form

$$y' = f(x, y)$$

is **homogeneous** in x and y means that

for all λ in some interval. To solve such an equation, let ν be such that

The result will be a

. Solve for v then find y.

Note: In this section of the text these equations are simply called homogeneous. The term homogeneous usually refers to a linear equation where the right side is zero. That is the usage you find beginning in Chapter 3.

Example. Find a parameterized family of solutions (or find the general solution) to

$$y' = \frac{x^2 + y^2}{2xy}$$
 on intervals of positive numbers.

Solution. The differential equation is of the form

$$v' = f(x, v)$$

where

$$f(x,y) =$$

Note that

$$f(\lambda x, \lambda y) = =$$

So the differential equation is homogeneous

Ley v be such that xv = y. Then from the differential equation we have

=

so

 $= \cdot$

= ·

or

From this we get

= · ...

Thus

so =

=

so _

SO

$$|1-v^2|=C_3\frac{1}{x}$$

where $C_3 = e^{c_2}$. Since xv = y we have

$$|1 - \frac{y^2}{x^2}| = C_3 \frac{1}{x}$$

= (,

For solutions where $x^2 \ge y^2$ this becomes

=

where $C = -C_3$. For solutions where $x^2 < y^2$ this becomes

 $\dot{}$ = (

where $C = C_3$.

Additional Examples. See those in Section 2.3 of the text and those presented on the board in class.

Suggested problems for Section 2.3. Do the odd numbers 1 through 25.