Section 2.3

Section 2.3

Two More First Order Differential Equations

Definition. A differential equation that is equivalent to one of the form

$$y' + p(x)y = q(x)y^r$$

where $r \neq 0$ and $r \neq 1$ is said to be a **Bernoulli** differential equation.

When the equation is in this form, it can be solved

The result is a first order linear differential equation for v.

find y.

Example. Find all solutions to

$$y' + \frac{1}{x}y = 3x^2y^2$$ on intervals of positive numbers.

Solution. Multiply each side by to get

Let

so

or

The integrating factor for this first order linear d.e. is

so

or

$$(\frac{1}{x}v)' = -3x.$$

$y')$

Integrating we have

so

Since

it follows that

or

The C in the last equation is twice the C in the previous equation.
is a singular solution.

Saying that a differential equation of the form

$$y' = f(x,y)$$

is **homogeneous** in x and y means that

for all λ in some interval. To solve such an equation, let v be such that

The result will be a . Solve for v then find y.

Note: In this section of the text these equations are simply called homogeneous. The term homogeneous usually refers to a linear equation where the right side is zero. That is the usage you find beginning in Chapter 3.

Example. Find a parameterized family of solutions (or find the general solution) to

$$y' = \frac{x^2 + y^2}{2xy}$$ on intervals of positive numbers.

Solution. The differential equation is of the form

$$y' = f(x,y)$$

where

$$f(x,y) =$$

Note that

$$f(\lambda x, \lambda y) = \quad = \quad = \quad .$$

So the differential equation is homogeneous

Ley v be such that $xv = y$. Then from the differential equation we have

$$=$$

so

$$=$$

or

$$=$$

or

$$= \quad .$$

From this we get

$$=$$

so

$$= \quad .$$

Thus

$$=$$

so

$$=$$

so

$$=$$

so

$$|1 - v^2| = C_3 \frac{1}{x}$$

where $C_3 = e^{c_2}$. Since $xv = y$ we have

$$|1 - \frac{y^2}{x^2}| = C_3 \frac{1}{x}$$

or

$$= (\quad ,$$

For solutions where $x^2 \geq y^2$ this becomes

$$=$$

where $C = -C_3$. For solutions where $x^2 < y^2$this becomes

$$= ($$

where $C = C_3$.

Additional Examples. See those in Section 2.3 of the text and those presented on the board in class.

Suggested problems for Section 2.3. Do the odd numbers 1 through 25.