Section 2.4.1

Orthogonal means

Lines in the plane are perpendicular or orthogonal if and only if

or

Smooth curves in the plane are orthogonal at a point of intersection

Suppose that $\mathcal{F}$ is a one-parameter family of smooth curves in the plane. Saying that $\mathcal{G}$ is the family of **orthogonal trajectories** for $\mathcal{F}$ means that $\mathcal{G}$ is a one-parameter family of smooth curves in the plane and

Example.

Procedure. To find the orthogonal trajectories for a given family,

and

Example. Find the orthogonal trajectories for the family of parabolas indicated by
$$x = Cy^2. \qquad\qquad 1$$

Solution. From (1) we have

2

Solving (1) for C to get

and putting this value of C into (2) we have

so

3

This is the d.e. for the given family.

3

Replacing y' with $-1/y'$ we have

or

4

This is the d.e. for the orthogonal trajectories. It is separable. Solving it by integration we have

or

5

(5) gives the orthogonal trajectories. It is

Additional Examples: See Section 2.4.1 of the text and the notes presented on the board in class.

Suggested Problems. Do the odd numbered problems for section 2.4.1. The answers are posted on Dr. Walker's web site.