Notes on Section 2.4.4
Falling Objects with Air Resistance

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Suppose that an object of mass m falls under gravity near the surface of the earth with air rresistance proportional to its velocity. Then from second law of motion we have

$$mv'(t) = -mg - kv(t) \text{ for } t \geq 0$$

where $v(t)$ is the velocity of the object at time t, g is the magnitute of the acceleration due to gravity, and k is a positive constant known as the drag coefficient. The equation is equivalent to

$$mv'(t) + kv(t) = -mg \text{ for } t \geq 0$$

which is a first order linear equation. Solving it, you should find that v is a solution if and only if

$$v(t) = ce^{-kt/m} - \frac{gm}{k}$$

for some number c and all $t \geq 0$.If $v(0) = v_0$,then

$$v(t) = (v_0 + \frac{gm}{k})e^{-kt/m} - \frac{gm}{k}$$

The altitude $y(t)$ at time t is related to the velocity by

$$v(t) = y'(t) \text{ for } t \geq 0$$

So

$$y(t) - y(t_0) = \int_{t_0}^{t} v(\tau)d\tau.$$

Suggested Problem. Problem 1 in Exercises 2.4.4 on page 51 of the text.