Notes on Section 2.4.5
Mixing Problems

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A fluid containing a substance (e.g. a salt water solution) with concentration k_1 flows into a mixing container at a rate of R_1 and is mixed with the fluid already there. . Let

and let

The rate of change in A is the

The inflow rate of the substance is . The concentration of the substance in the container at time t is . This is also the concentration of the substance in the fluid that is being pumped out. Thus the outflow rate of the substance is

Putting everything together, we have

for $t \geq 0$. (1)

Note that V is constant when and

$$V(t) =$$

when $R_1 \neq R_2$.

Note that (1) is equivalent to

(2)

which is a

Example. A 100 gallon tank is initially full of water. At time $t = 0$ a 20% hydrochloric acid solution begins to flow into the tank at a rate of 2 gallons/minute. The well-mixed solution in the tank is pumped out at the same rate. Find the amount $A(t)$ (in gallons) of acid in the tank at time t.

Solution. Note that V . Thus (2) becomes

which becomes

Using the integrating factor we have

Defining B by

the last differential equation becomes

so

Thus

for some number C and all $t \geq 0$. Since the mixing tank contains only water when $t = 0$, we have

Thus and

Example. See the example on pages 52 and 53 of the text.

Suggested Problems. Problems 1, 3, and 5, in Exercises 2.4.5 on pages 53 and 54 of the text.