

# Notes on Section 2.4.6 The Logistic Equation

Philip W. Walker

The differential equation

$$\boxed{\phantom{y' + ky = M}} \tag{1}$$

where each of  $k$  and  $M$  is a positive number is known as the  $\boxed{\phantom{\text{logistic equation}}}$   
The equation has applications in mathematical biology. For example,  $\boxed{\phantom{\text{population growth}}}$

$\boxed{\phantom{\text{population growth}}}$  The number  $\boxed{\phantom{\text{population growth}}}$   
and  $\boxed{\phantom{\text{population growth}}}$

Equation (1) is equivalent to

$$\boxed{\phantom{y' + ky = M}}$$

which is a Bernoulli differential equation. It is also equivalent to

$$\boxed{\phantom{y' + ky = M}} \tag{2}$$

which is separable. We will find the solutions to (1) by solving (2). Using partial fractions

$$\boxed{\phantom{\frac{1}{y(M-y)} = \frac{1}{My} + \frac{1}{y(M-y)}}}$$

on the left side of (2) we have

$$\boxed{\phantom{\frac{1}{y(M-y)} = \frac{1}{My} + \frac{1}{y(M-y)} + C}}$$

In the applications of interest it will be the case that  $0 < y(t) < M$  so

$$\boxed{\phantom{\frac{1}{y(M-y)} = \frac{1}{My} + \frac{1}{y(M-y)}}}$$

so

$$\boxed{\phantom{\frac{1}{y(M-y)} = \frac{1}{My} + \frac{1}{y(M-y)}}}$$

Exponentiating and noting that the additive constant becomes a multiplicative one,

$$\boxed{\phantom{y = \frac{M}{1 + Ce^{-kt}}}}$$

so



If



where  $0 < R < M$  then



**Example.** See the example on page 56 of the text.

**Suggested Problems.** Problems 1,3, and 5 in Exercises 2.4.6 on pages 56 and 57 of the text.