# Notes on Section 2.4.7

Philip W. Walker

The following are solutions and solution suggestions for some of the problems in this subsection.

1. A disease is infecting a colony of 1000 penguins living on a remote island Let $P(t)$ be the number of sick penguins $t$ days after the outbreak. Suppose that 50 penguins had the disease initially, and suppose that the disease is spreading at a rate proportional to the product of the time elapsed and the number of penguins who do not have the disease.

   (a) The model is

   and

   (b) The differential equation is separable.

   is the general solution to the differential equation.

   (c) The initial condition is

   so

   is the solution to the initial value problem.

2. A disease is infecting a herd of 100 cows. Let $P(t)$ be the number of sick cows $t$ days after the outbreak. Suppose that 15 cows had the disease initially, and suppose that the disease is spreading at a rate proportional to the number of cows who do not have the disease.

   (a) The model is

   and

   (b) The differential equation is separable.

   is the general solution.

   (c)

   so

   Thus

   and

   This is the solution to the initial value problem.

3. A 1000-gallon tank, initially full of water, develops a leak at the bottom. Given that 200 gallons of water leaks out in the first 10 minutes, find the amount of water, $A(t)$ in the tank $t$ minutes after the leak develops if water drains off at a rate proportional to the product of the time elapsed and the amount of water present.

**Solution.**

Solving the d.e. we have

applying $A(0) = 1000$ we have

and

. (*)

Using $A(10) = 800$ we have

so

and

Putting this value for $k$ into (*) we have

=

so

4. A 1000-gallon tank, initially full of water, develops a leak at the bottom. Given that 500 gallons of water leaks out in the first 30 minutes, find the amount of water, $A(t)$ in the tank $t$ minutes after the leak develops if water drains off at a rate proportional to the product of the time elapsed and the square root of the amount of water present.

**Solution.**

Solving the differential equation, we have

$A(0) = 1000$ so

so

Continuing,

(*)

$A(30) = 500$ s0

From (*),

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So

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5. A 1000-gallon tank, initially full of water, develops a leak at the bottom. Given that 300 gallons of water leaks out in the first 20 minutes, find the amount of water, $A(t)$ in the tank $t$ minutes after the leak develops if water drains off at a rate proportional to the square of the amount of water in the tank.

**Solution.**

Solving the differential equation we have

.

Using $A(0) = 1000$ this becomes

so

Using $A(20) = 700$ we have

so

and

So

6. Let $P(t)$ be be the number (in millions) of people who become aware of the product by time $t$.