Section 3.4

Section 3.4 Nonhomogeneous Second Order Linear Differential Equations Part 1

In this section and the next, we will be concerned with finding the solutions to the nonhomogeneous equation	
	Ν
on an interval J when each of p , q , and f is a continuous function with domain J . In order to solve (N) we will first need to solve the	
	Н
which is sometimes called the $\cite{Mathemath{In}}$ In connection with (N) and (H) we define the linear differential operator L by	
whenever y is a twice differentiable function with domain J .	
Recall that	
$L(c_1y_1+c_2y_2)=$	
Consequently,	
and <i>I</i>	
Also, if	
and then	
The	
THE	
	N
	Н
In order to find all solutions to the nonhomogeneous equation (N) we need	
(called a) and {	
).	
Theorem. Suppose that	

(The function z is called a to the nonhomogeneous equation (N), and the following is a description of (N) .) It follows that
if and only if
for some <i>u</i> such that
which means
Proof. If $Ly = f$, let and = on J .
If and on J, then = _ =
If $\{y_1,y_2\}$ is a fundamental pair or set for (H), the u in the last theorem can be replaced with
Theorem . Suppose that $\{y_1,y_2\}$ is a fundamental pair or set for L , and
Lz = f on J .
It follows that
if and only if
for
While the solutions to (H) are given by
when $\{y_1,y_2\}$ is a fundamental pair, we will see that a particular solution to (N) is of the form
when $\{y_1, y_2\}$ is a randomental pair, we will see that a particular solution to (iv) is of the form
where each of u and v is a function. Hence the name,
The following theorem gives a formula for a particular solution to the nonhomogeneous
equation.
Theorem . Suppose that $\{y_1,y_2\}$ is a fundamental set for(H). Let W be the Wronskian of
(y_1,y_2) and let
z(x) =
or

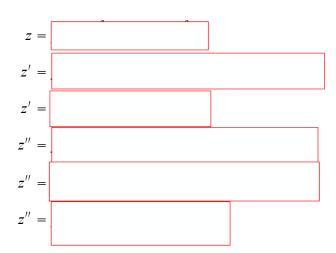
It follows that z is a particular solution to (N).

z(x) =

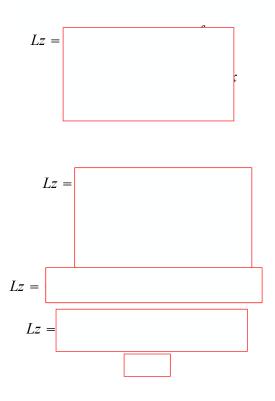
Note. Leave of the "+C" when finding the anti-derivatives. The

formula is given in the text. The equivalent form (2) is easier to remember and

Proof.

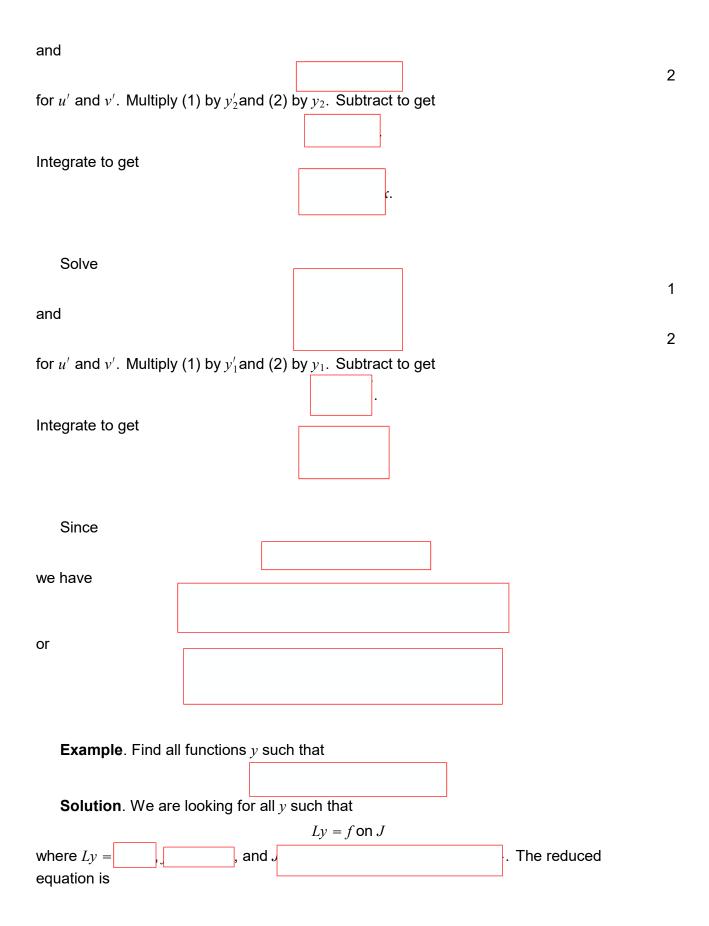


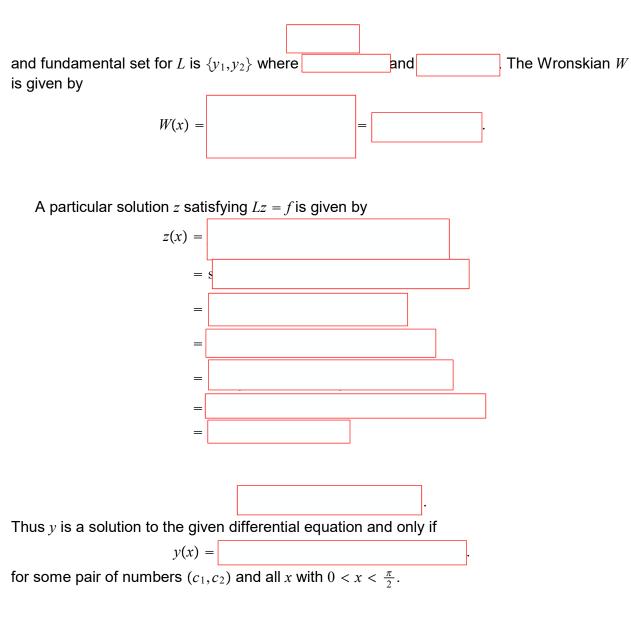




Note. If you want to derive the formula, start with

1





Note. Sometimes when applying this method, the solution z will turn out to be of the form

Example. Find all functions y such that

	for all x in \mathbb{R} .				
Solution The reduced	Solution The reduced equation is				
Column The reduced	= 0.				
The characteristic polynon					
The characteristic polynon					
so a fundamental pair is {	$\{y_1,y_2\}$ where $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$ and $\{y_2,y_2\}$ and $\{y_2,y_2\}$ and $\{y_1,y_2\}$				
	W(x) =				
A particular solution z s	satisfying the given nonhomogeneous equation is given by				
2	(x) =				
	= .				
Since					
the first integrand can be r	e-written so that				
	= -				
Note that					
where					
and					
Since z_2 is a linear combination	ation of y_1 and y_2 ,				

nd consequently,
Thus on ℝ
and only if
y(x) =
r some pair of numbers (c_1, c_2) and all real numbers x .
Additional Examples:
Suggested Problems.