Section 3.5
The Method of Undetermined Coefficients

Note. In this section, L will be a constant coefficient second order linear differential operator with

where each of a, and b is a real number. P will be the polynomial associated with or characteristic polynomial for L.

When L is a second order and f is of , the form of a particular z solution to the nonhomogeneous equation

can be predicted. This form will contain coefficients that must be determined in order to specify the solution.

Definition. Saying that r_0 is a zero of P means that

Saying that $r_0$is a zero of for P means that

$=$ where .

Saying that $r_0$is a zero of for P means that

Case 1. If

where (

, there will be a particular solution z satisfying

of the form

Example. Find all solutions to

Solution. Here so the zeros of P The number

. There will be a particular solution z of the form

$$Lz = z'' - z' - 6z =$$

$$=$$

This will be the right side of the given differential equation if and only if

or

Thus there is a particular solution z given by

Since the zeros of the associated polynomial are a fundamental pair for the related homogeneous or reduced equation is . Thus

.

if and only if

.

Case 2. If

where and (

), there will be a particular solution z satisfying

$$Lz = f$$

of the form

where The values

of the coefficients are found and after

Example.
Give the form of a particular solution to

Solution. Here The number . There will be a particular solution z of the form

Case 3. If

where and , let . . There will be a particular solution z satisfying

$$Lz = f$$

of the form

where

of the

Case 4. Suppose that each of α and β is a real number with $\beta \neq 0$ and

where each of . , . (Since L is second order with real coefficients, this will mean $m = 1$. Later when L is of higher order $m > 1$ will be possible.) There will be a particular solution z satisfying

i.e.

of the form

where

and,

The even if f contains only sine terms or only cosine terms.

Example. Find all solutions to

Solution. . Think of the right side of the differential equation as

and

so , , and

.

$$Lz = z'' - z' - 6z$$

$$=$$

$$=$$

This will be if and only if

and

or

or

Thus

and y is a solution to the given differential equation if and only if

Example. Find all solutions to

Solution. so P has only one zero, and it is of

The right side of the differential equation is

Thus a particular solution z will be of the form

so

Computation shows that

$$z' =$$

$$z'' =$$

and

$$Lx = z'' + 2z' + z = .$$

This will be the given

if and only if , and if and only if , , and . Thus

and y is a solution to the given differential equation if and only if

.

Note. If the method of undetermined coefficients can be used to solve

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for then it can be used to solve

and a particular solution will be of a form that is . Use predicted form.

Example. Give the form of a particular solution to

Solution.

Note that the so the

Note. If L is or if f is of the types considered above, the must be used to solve

Example.The method of undetermined coefficients cannot be used to solve

Additional Examples: See Section 3.5 of the text and the notes presented on the board in class.

Suggested Problems. Do the odd numbered problems for Section 3.5. The answers are posted on Dr. Walker's web site.