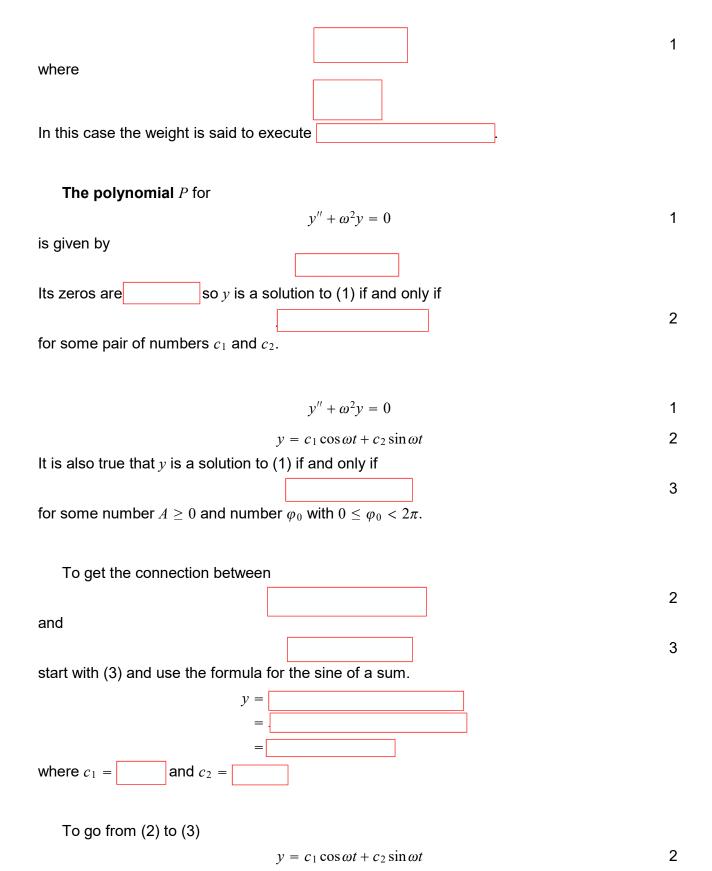
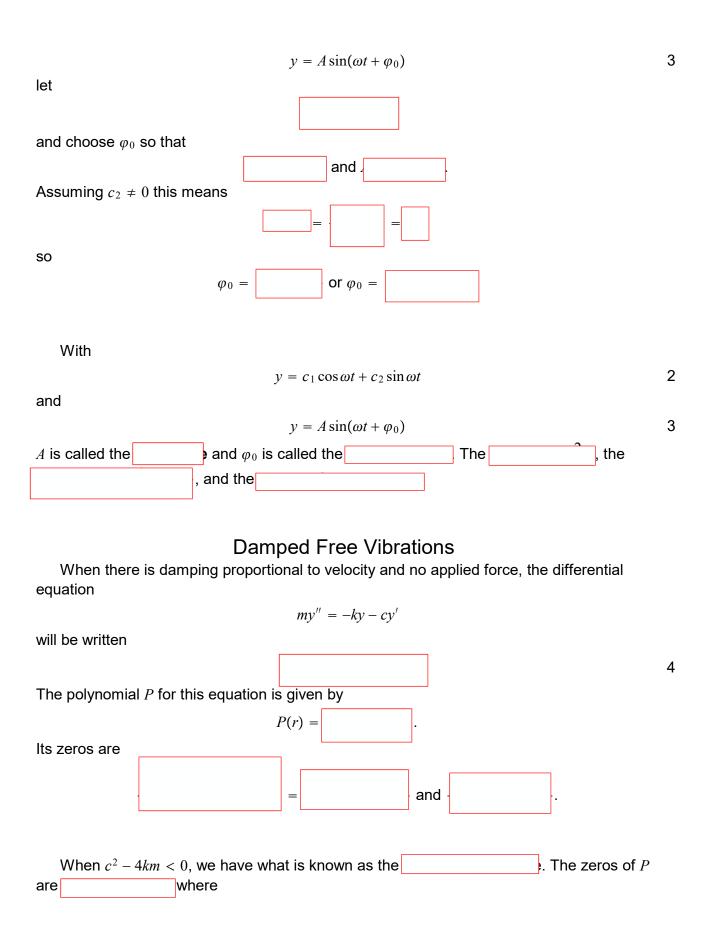
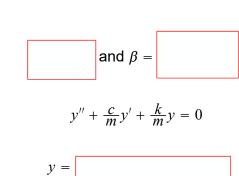
Section 3.6

Section 3.6 Vibrating Mechanical Systems

Suppose that a weight of mass	·	y a spring with s	pring const	ant k , and
the weight moves only in the vertical	,			
	Note that	and		
	Take the			and let the
magnitude of			_	when the
spring is				
. Using			we have	
Let y be the displacement of the we	eight from the equ	ilibrium position.		
Note that and . Since				
and . Since	-			
for equilibrium, we have		_		
my'' =		_		
my –				
-				
Thus				
If there is damping proportional to	elocity, the differ	ential equation be	ecomes	
	, , ,	¬		
where c is a positive constant. Then	re might also be a	n external force	applied. Fo	r example,
In this o	ase the differentia	al equation beco	mes	
I II		! 4!		
	mped Free V			
When there is no damping and	no applied force t	he differential ed	luation	
will be written				
AAIII DO AAIIIIOII				







if and only if

so y is a solution to

It is also true that y a solution to (4) if and only if

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 $y= \boxed{$ for some number A>0 and number φ_0 with $0\leq \varphi_0<2\pi.$ Note that since c>0, it follows that $\alpha < 0$. but with

When $c^2 - 4km = 0$, we have what is known as the ase. P has , so y is a solution to only one zero,

$$y'' + \frac{c}{m}y' + \frac{k}{m}y = 0$$

if and only if

for some pair of numbers c_1 and c_2 . Since $r_0 < 0$ it follows that

$$\lim_{t\to\infty}y(t)=0.$$

When $c^2 - 4km > 0$, we have what is known as the case. P has two zeros, $r_1 = -$ and $r_2 =$

so y is a solution to

$$y'' + \frac{c}{m}y' + \frac{k}{m}y = 0$$

if and only if

$$y =$$
 it follows that

$$\lim_{t\to\infty}y(t)=0.$$

Undamped Forced Vibrations

We consider next the case where there is no damping and a sinusoidal external applied force. We will take

G(t) =

so that

my'' = -ky + G(t)

becomes

where

The method of undetermined coefficients can be used to find a particular solution to (5). We found all solutions to the reduced equation when we considered undamped free vibrations.

 $\gamma/2\pi$ is called the and $\omega/2\pi$ is called the . When is a solution to

$$y'' + \omega^2 y = \frac{F_0}{m} \cos \gamma t$$
 5

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if and only if

for some pair of numbers c_1 and c_2 . In this case, it is also true that y is a solution to (5) if and only if

for some A>0 and φ_0 with $0\leq \varphi_0<2\pi$. In this case the oscillations are bounded but large in magnitude if γ is close to ω .

When $\gamma=\omega,$ we have a situation known as y is a solution to if and only if y=

for some pair of numbers c_1 and c_2 . In this case, it is also true that y is a solution to (5) if and only if



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for some A>0 and φ_0 with $0\leq \varphi_0<2\pi.$ The oscillations increase in magnitude without bound as t increases.

Damped Forced Vibrations

We consider next the case where there is damping and a sinusoidal external applied force so that

$$my'' = -ky - cy' + G(t)$$

becomes

where

The method of undetermined coefficients can be used to show that a particular solution z to (12) is given by

$$z(t) = -\frac{1}{2}$$

which can also be expressed by

$$z(t) =$$

So y is a solution to (12) if and only if

where u is a solution to the related homogeneous or reduced equation

Resonance also occurs in this case. The amplidude of the steady state solution z given by

$$z(t) = \frac{1}{2}$$

is at a maximum when $\gamma = \omega$ where it becomes



Fach such solution u to the related homogeneous or reduced equation is called a and the particular solution z is called the

We have seen in the Damped Free Vibration case that

$$\lim_{t\to\infty}y(t)=0$$

for each solution to

$$y'' + cy' + \omega^2 y = 0,$$

hence the name transient solution in this case.

Additional Examples: See Section 3.6 of the text and the notes presented on the board in class.

Suggested Problems. Do the odd numbered problems for Section 3.6. The answers are posted on Dr. Walker's web site.