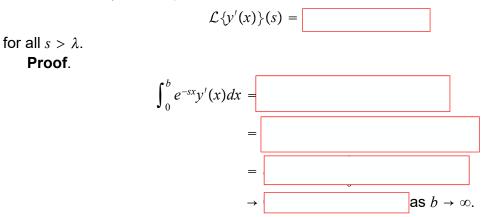
Section 4.2

## Section 4.2 The Laplace Transform - Basic Properties

Definition. Supp	ose that $f$ is a function with domain $[0,\infty)$ and $\lambda$ is a real number. Saying
that	means that there is aand a
	such that
	for all $x \ge A$ .
Saying that <i>f</i> is of exponential order means that <i>f</i> is of exponential order $\lambda$ for some number	
λ.	
<b>Note</b> . If $f(x) =$	
	, ,
	or
where	$\square$ and each of <i>r</i> and $\beta$ is a real number, then <i>f</i> is of exponential order.
Evenue la	
Example. If	
then <i>f</i> is not of exponential order. $f(x)$ grows too fast as $x \to \infty$ .	
Theorem.	, then
meorem.	
	$\mathcal{L}{f(x)}(s)$
exists for all $s > \lambda$ .	



**Theorem**. If *y* is of exponential order  $\lambda$  and has a continuous derivative on  $[0,\infty)$  then



Proof.

**Corollary**. If *y* has a continuous *n*-th derivative and each of *y*, y', ..., and  $y^{(n-1)}$  is or exponential order  $\lambda$ , then

$$\mathcal{L}\{y^{(n)}(x)\}(s) =$$

Note. If

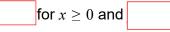
 $\mathcal{L}\{y'(x)\}(s) =$  $\mathcal{L}\{y''(x)\}(s) =$ 

 $Y(s) = \mathcal{L}\{y(x)\}(s)$ 

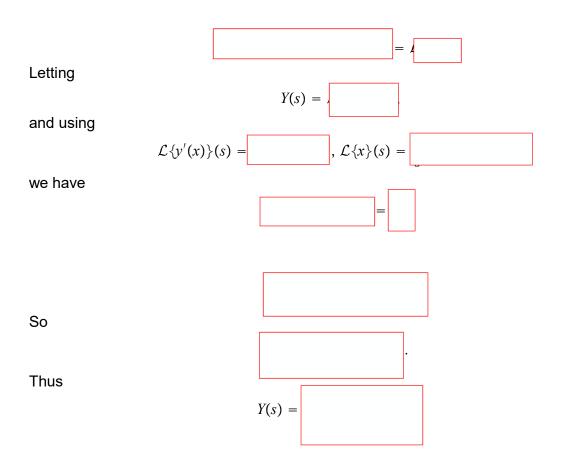
and

then

Example. Find the Laplace transform of the solution to the following initial value problem.



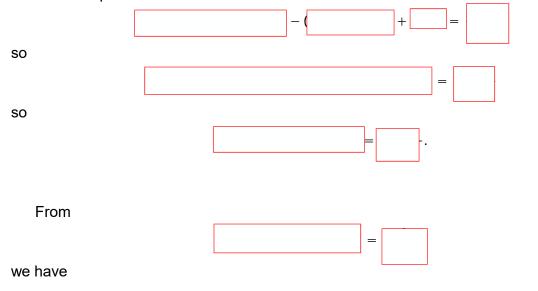
Solution. Taking the Laplace transform of each side of the differential equation and using the fact that the transform is linear we have



**Example**. Find the Laplace transform of the solution to the following initial value problem.



**Solution**. Letting  $Y(s) = \mathcal{L}{y(x)}(s)$  and taking the Laplace transform of each side of the differential equation we have





The next theorem shows how to find the Laplace transform of  $e^{rx}f(x)$  if the transform of f(x) can be found.

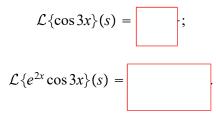
Theorem. If

$$\mathcal{L}{f(x)}(s) = F(s)$$
 for  $s > s_0$ 

then

$$\int e^{rx} f(x) = F(s-r) \text{ for } s > s_0 + r$$

Example.



so

The next theorem shows how to find the Laplace transform of  $x^n f(x)$  if the transform of f(x) can be found.

Theorem. If

 $\mathcal{L}{f(x)}(s) = F(s)$ 

$$\mathcal{L}\{x^n f(x)\}(s) =$$

$$\mathcal{L}\{\sin 3x\}(s) = F(s)$$

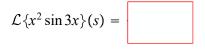
where

then

$$F(s) =$$
 .  
 $F'(s) =$  - .  
 $F''(s) = \cdots =$ 

and

SO



Additional Examples: See Section 4.2 of the text and the notes presented on the board in class.

**Suggested Problems**. Do the odd numbered problems for Section 4.2. The answers are posted on Dr. Walker's web site.