Section 4.3

Definition. The inverse Laplace transform is denoted by $\mathcal{L}^{-1}$.

means .

Note. Based on the transforms that we know, we have the following.

Note. Since the transform is , the inverse transform is also

Note. In order when in the denominator, .

Example.

$$\frac{1}{s^2 + 7s + 12} =$$

Using partial fractions we have

$$\frac{1}{(s+3)(s+4)} =$$

Adding the fractions, we have

$$\frac{1}{(s+3)(s+4)} =$$

Equating the numerators, we have

1

Letting we have

Letting we have

Thus

$$\frac{1}{(s+3)(s+4)} =$$

and

$$\mathcal{L}^{-1}\{\frac{1}{(s+3)(s+4)}\}(x) =$$

$$=$$

Example.

$$\mathcal{L}^{-1}\{\frac{s+3}{s^2+4s+13}\}(x) = \mathcal{L}^{-1}\{\quad\}(x)$$

$$= \mathcal{L}^{-1}\{\quad\}(x)$$

$$= \quad \mathcal{L}^{-1}\quad\}(x)$$

$$= \quad + $$

$$= \quad + \quad)$$

$$=$$

Example. Use the Laplace transform to solve the initial value problem:

for $x \geq 0$ and .

Solution. Taking the Laplace transform of each side of the differential equation and letting $Y(s) = \mathcal{L}\{y(x)\}(s)$, we have

$$=$$

so

$$= \quad .$$

So

$$Y(s) =$$

so

$$Y(s) = \quad .$$

Using partial fractions, we have

$$\frac{5s^2+92}{(s+2)(s^2+16)} = \quad = \quad .$$

Equating numerators, we have

Letting (2) produces

so

Letting) now produces

so

Letting now produces

so

$$Y(s) =$$

$$=$$

$$=$$

$$=$$

So

$$y(x) =$$

Example. Use the Laplace transform to solve the initial value problem:

for $x \geq 0$,

Solution. Taking the Laplace transform of each side of the differential equation and letting $Y(s) = \mathcal{L}\{y(x)\}(s)$, we have

$$+ 4 \quad + 6 \quad =$$

$$(s^2 + 4s + 6)Y(s) = \frac{2s + 1}{s(s + 1)}$$

$$Y(s) = \quad .$$

Using partial fractions, we find that

$$Y(s) =$$

so

$$Y(s) =$$

Continuing we find that

$$Y(s) =$$

so

$$Y(s) =$$

Thus

$$y(x) =$$

Additional Examples: See Section 4.3 of the text and the notes presented on the board in class.

Suggested Problems. Do the odd numbered problems for Section 4.3. The answers are posted on Dr. Walker's web site.