Section 4.5
Piecewise Defined and Piecewise Continuous Functions
Part II - Inverse Laplace Transforms

The main theorem for this section is the following.

Theorem.

$$\mathcal{L}^{-1}\{e^{-as}F(s)\}(x) = u(x-a)\mathcal{L}^{-1}\{F(s)\}(x-a)$$

$\mathcal{L}^{-1}\{F(s)\}(x-a)$ means the inverse Laplace transform of $F(s)$ evaluated at $(x-a)$ rather than x.

Example.

$$\begin{aligned}\mathcal{L}^{-1}\{e^{-3s}\frac{2}{s^3}\}(x) &= u(x-3)\mathcal{L}^{-1}\{\frac{2}{s^3}\}(x-3)\\ &= u(x-3)(x-3)^2\\ &= \begin{cases} 0 & if \;\; x<3\\ (x-3)^2 & if \;\; x\ge 3\end{cases}\end{aligned}$$

Example.

$$\begin{aligned}\mathcal{L}^{-1}\{\frac{se^{-s}}{s^2+\pi^2}\}(x) &= u(x-1)\mathcal{L}^{-1}\{\frac{s}{s^2+\pi^2}\}(x-1)\\ &= u(x-1)\cos(\pi(x-1))\\ &= \begin{cases} 0 & if \;\; x<1\\ \cos(\pi(x-1)) & if \;\; x\ge 1\end{cases}\end{aligned}$$

Example.

$$\mathcal{L}^{-1}\{\frac{2}{s^2} + \frac{(s+2)e^{-2s}}{s^3} - \frac{4e^{-3s}}{s}\}(x)$$

$$
\begin{aligned}
&= \mathcal{L}^{-1}\{\frac{2}{s^2}\}(x) \\
&\quad + \mathcal{L}^{-1}\{\frac{(s+2)e^{-2s}}{s^3}\}(x) - \mathcal{L}^{-1}\{\frac{4e^{-3s}}{s}\}(x) \\
&= \mathcal{L}^{-1}\{\frac{2}{s^2}\}(x) \\
&\quad + u(x-2)\mathcal{L}^{-1}\{\frac{(s+2)}{s^3}\}(x-2) - \mathcal{L}^{-1}\{\frac{4e^{-3s}}{s}\}(x) \\
&= 2x + u(x-2)(x-2+(x-2)^2) - 4u(x-3) \\
&= \begin{cases} 2x & \textit{if} \quad x < 2 \\ 3x - 2 + (x-2)^2 & \textit{if} \quad 2 \le x < 3 \\ 3x - 6 + (x-2)^2 & \textit{if} \quad 3 \le x \end{cases}
\end{aligned}
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Additional Examples. See Section 4.5 of the text and the notes presented on the board in class.

Suggested Problems. Do the odd numbered problems for Section 4.5. The answers are posted on Dr. Walker's web site.