## Probability Formulas

- $\operatorname{Pr}(E \cup F)=\operatorname{Pr}(E)+\operatorname{Pr}(F)-\operatorname{Pr}(E \cap F)$
- $\operatorname{Pr}(E \mid F)=\frac{\operatorname{Pr}(E \cap F)}{\operatorname{Pr}(F)}$
- $\operatorname{Pr}(E)=\sum_{i=1}^{n} \operatorname{Pr}\left(E \mid F_{i}\right) \operatorname{Pr}\left(F_{i}\right)$ whenever $F_{1}, \ldots, F_{n}$ is a partition of the sample space.
- $\operatorname{Pr}(F \mid E)=\frac{\operatorname{Pr}(E \mid F) \operatorname{Pr}(F)}{\operatorname{Pr}(E)}$
- $\sum_{k=0}^{n} r^{k}=\frac{1-r^{n+1}}{1-r}$ when $r \neq 1$
- $\sum_{k=0}^{\infty} r^{k}=\frac{1}{1-r}$ when $-1<r<1$
- When $f$ is the bivariate probability density or probability mass function for the pair of random variables $(X, Y)$

$$
\begin{gathered}
f_{X}(x)=\sum_{y} f(x, y) \text { in the discrete case } \\
f_{X}(x)=\int_{-\infty}^{\infty} f(x, y) d y \text { in the continuous case } \\
f_{Y}(y)=\sum_{x} f(x, y) \text { in the discrete case } \\
f_{Y}(y)=\int_{-\infty}^{\infty} f(x, y) d x \text { in the continuous case } \\
f_{X}(x \mid Y=y)=\frac{f(x, y)}{f_{Y}(y)} \\
f_{Y}(y \mid X=x)=\frac{f(x, y)}{f_{X}(x)} \\
f_{X}(x)=\sum_{y} f_{X}(x \mid Y=y) f_{Y}(y) \text { in the discrete case } \\
f_{X}(x)=\int_{-\infty}^{\infty} f_{X}(x \mid Y=t) f_{Y}(t) d t \text { in the continuous case } \\
f_{Y}(y)=\sum_{x} f_{Y}(y \mid X=x) f_{X}(x) \text { in the discrete case } \\
f_{Y}(y)=\int_{-\infty}^{\infty} f_{Y}(y \mid X=t) f_{X}(t) d t \text { in the continuous case } \\
f_{Y}(y \mid X=x)=\frac{f_{X}(x \mid Y=y) f_{Y}(y)}{f_{X}(x)}
\end{gathered}
$$

- Bernoulli.

$$
f(0)=q \text { and } f(1)=p
$$

where $0 \leq p \leq 1$ and $q=1-p$.

- Binomial.

$$
f(k)=\binom{n}{k} p^{k} q^{n-k} \text { for } k=0,1, \ldots, n
$$

where $n$ is a positive integer, $0 \leq p \leq 1$ and $q=1-p$.

- Poisson.

$$
f(k)=\frac{e^{-\lambda} \lambda^{k}}{k!} \text { for } k=0,1, \ldots
$$

where $\lambda$ is a positive real number.

- Geometric.

$$
f(k)=p q^{k} \text { for } k=0,1, \ldots
$$

where $0 \leq p \leq 1$ and $q=1-p$.

- Negative binomial.

$$
f(k)=\binom{r+k-1}{r-1} p^{r} q^{k} \text { for } k=0,1, \ldots
$$

where $r$ is a positive integer, $0 \leq p \leq 1$ and $q=1-p$.

- Uniform.

$$
f(x)=\frac{1}{b-a} \text { for } a \leq x \leq b
$$

where $a$ and $b$ are real numbers with $a<b$.

- Normal.

$$
f(x)=\frac{1}{\sqrt{2 \pi} \sigma} \exp \left(-\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right) \text { for all real numbers }
$$

where $\mu$ and $\sigma$ are real numbers with $\sigma>0$. When $\mu=0$ and $\sigma=1$, the cumulative distribution function is denoted by $\Phi$.

- Exponential.

$$
f(x)=\lambda e^{-\lambda x} \text { for } x \geq 0
$$

where $\lambda$ is a positive real number.

- Gamma

$$
f(x)=\frac{\lambda^{r} x^{r-1} e^{-\lambda x}}{\Gamma(r)} \text { for } x \geq 0
$$

where each of $r$ and $\lambda$ is a positive real number.

