The Gibbs Phenomenon

Suppose that f is piecewise smooth on $[-L, L]$ and f is discontinuous and/or $f(-L) \neq f(L)$. Let h be the limit of the Fourier Series for f. Near each jump in the graph of h, each S_n in the Fourier series will overshoot the graph of h. Moreover the magnitude of the overshoot does not diminish to zero as $n \to \infty$. It has limit l where l is approximately 9% of the magnitude of the jump.

Graphs of S_n when

$$f(x) = \begin{cases} 1 & \text{on } (0, \pi) \\ -1 & \text{on } (-\pi, 0) \end{cases}$$

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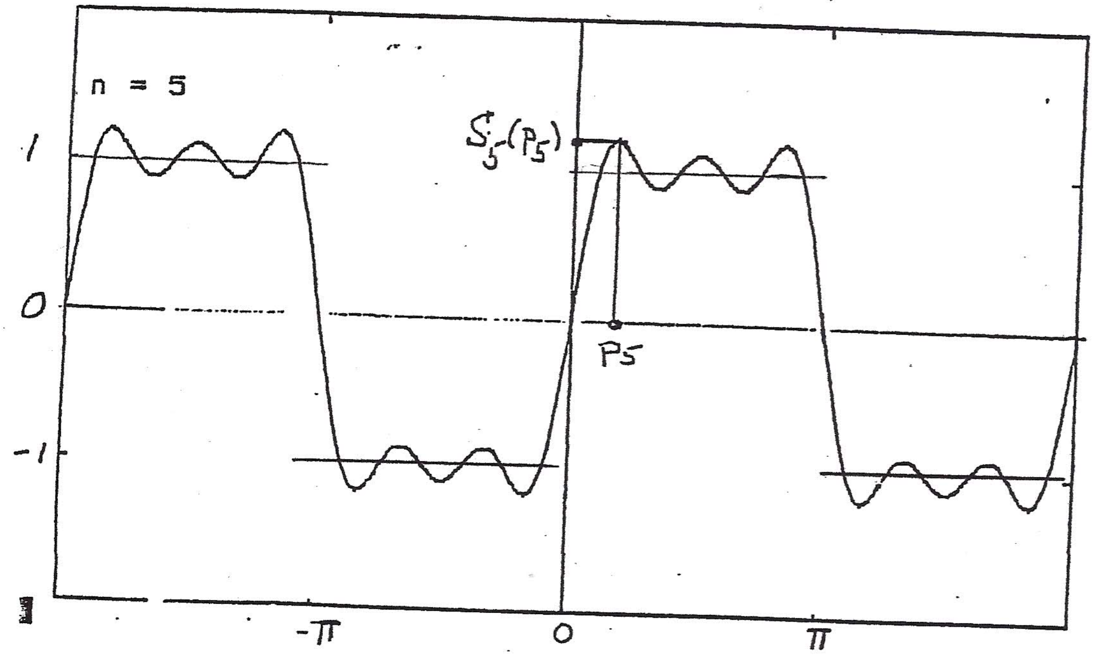

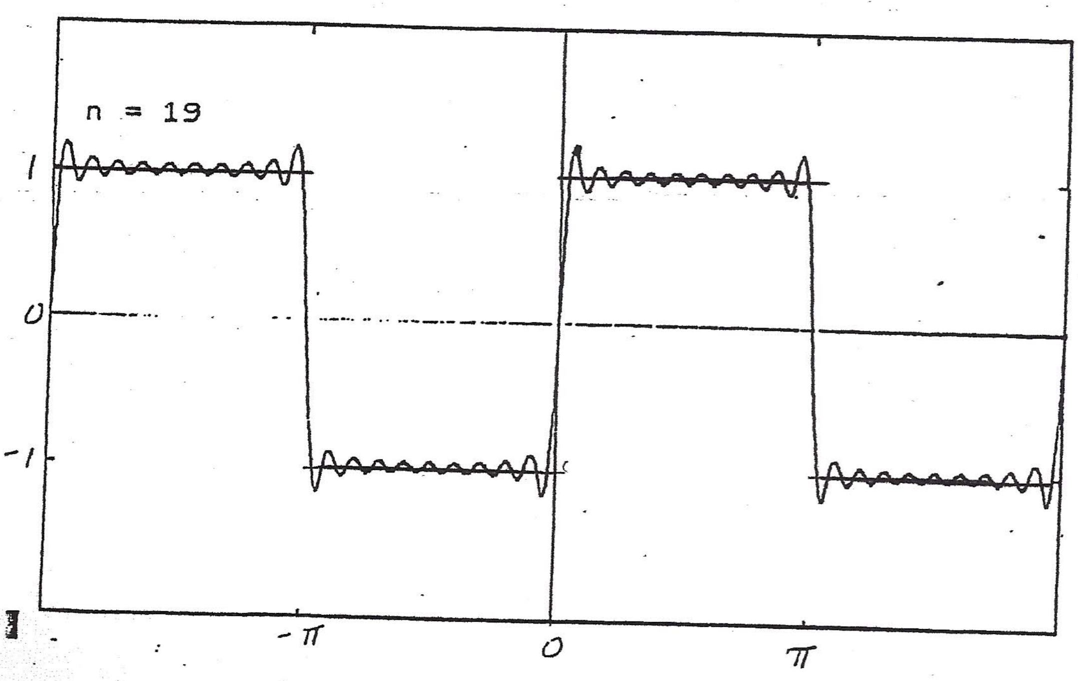

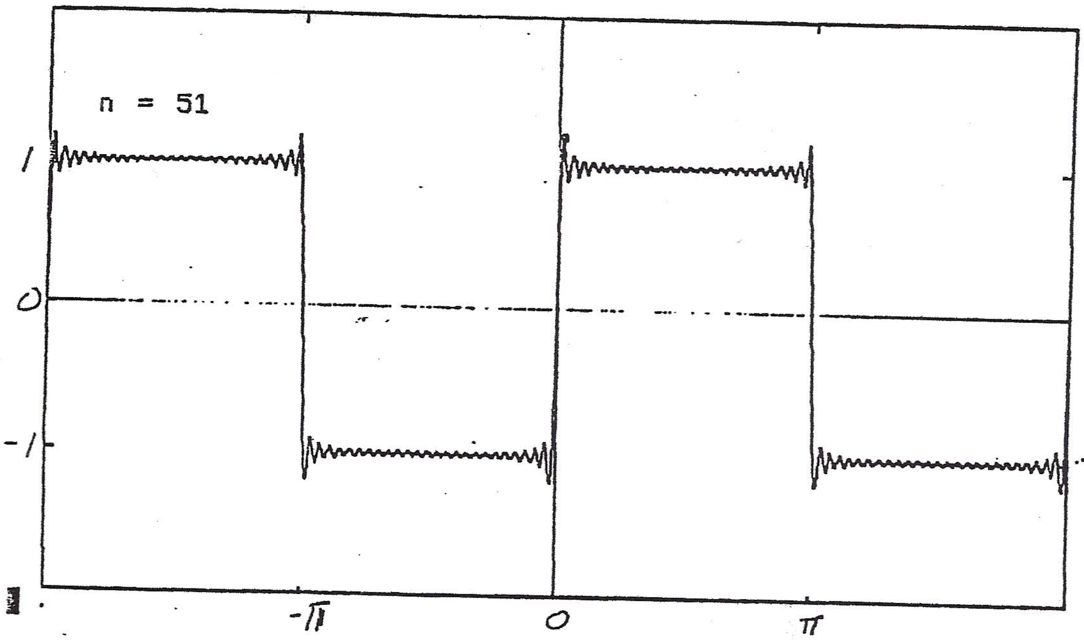

Graph of S_n when $f(x) = \begin{cases} 1 & \text{on } (0,\pi) \\ -1 & \text{on } (-\pi,0) \end{cases}$

$$S_n(x) = \frac{2}{\pi} \sum_{k=1}^{n} \frac{1-(-1)^k}{k} \sin kx .$$