

Laplace Equation Problem IV

PROBLEM: Derive the solution to

$$\frac{\partial^2 u}{\partial x^2}(x, y) + \frac{\partial^2 u}{\partial y^2}(x, y) = 0 \text{ for } 0 \leq x \leq L \text{ and } 0 \leq y \leq H, \quad (1)$$

$$u(0, y) = 0 \text{ for } 0 \leq y \leq H, \quad (2)$$

$$u(L, y) = f(y) \text{ for } 0 \leq y \leq H, \quad (3)$$

$$u(x, H) = 0 \text{ for } 0 \leq x \leq L, \text{ and} \quad (4)$$

$$u(x, 0) = 0 \text{ for } 0 \leq x \leq L. \quad (5)$$

Where each of H and L is a positive number.

SOLUTION: This problem is the same as Laplace Equation Problem II except that the roles of x and y have been interchanged and the roles L and H have been interchanged. The solution u to this problem is given by

$$u(x, y) = \sum_{k=1}^{\infty} E_k \sin \frac{k\pi y}{H} \sinh \frac{k\pi x}{H}$$

where

$$E_k = \frac{2}{H \sinh \frac{k\pi L}{H}} \int_0^H f(y) \sin \frac{k\pi y}{H} dy.$$