



Department of Mathematics
University of Houston
Numerical Analysis I
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Numerical Mathematics I (10. Homework Assignment)

Exercise 34 (L^2 -estimate for piecewise linear interpolation)

The interpolation error estimates presented in class provide upper bounds for the pointwise error in polynomial interpolation. Often, it is of interest to have estimates of the error in specific norms as, for instance, the L^2 -norm

$$\|f\|_{L^2([a,b])} := \left(\int_a^b |f(x)|^2 dx \right)^{1/2} .$$

Let $\Delta_h := \{x_i := ih\}_{i=0}^n$, $h := b/n$, $n \in \mathbf{N}$ be a uniform partition of the interval $[0, b]$ ($b > 0$). For $f \in C^2([0, b])$ we want to estimate the L^2 -norm of the interpolation error $f - s$, where s represents the polygon which interpolates f at the nodes x_i , $0 \leq i \leq n$.

(i) Show that for $x \in [0, h]$ there holds

$$|f(x) - s(x)| \leq \sqrt{3}x\sqrt{h} \left(\int_0^h |f''(t)|^2 dt \right)^{1/2} .$$

[Hinweis: Use the integral representation of the remainder in the Taylor expansion, the Cauchy Schwarz inequality, and the fact that for a continuously differentiable function Φ there holds

$$\Phi(x) - \Phi(0) = \int_0^x \Phi'(t) dt .]$$

(ii) Show that

$$\int_0^h |f(x) - s(x)|^2 dx \leq h^4 \int_0^h |f''(x)|^2 dx .$$

(iii) Prove the L^2 error estimate

$$\|f - s\|_{L^2([0,b])} \leq h^2 \|f''\|_{L^2([0,b])} .$$

Exercise 35 (*Uniform convergence of interpolating polynomials*)

Let $f(x) := \cos(x)$. For each $n \in \mathbb{N}$, assume that $x_i^{(n)} \in [0, \pi/2]$, $0 \leq i \leq n$, are pairwise different nodal points and denote by $p_n(f) \in P_n([0, \pi/2])$ the associated interpolating polynomial.

Prove that the sequence $\{p_n(f)\}_{\mathbb{N}}$ converges to f uniformly on $[0, \pi/2]$.

Exercise 36 (*Non-uniform convergence of interpolating polynomials*)

Let $f(x) = 1/(1+x^2)$. Show that for any $M > 0$ there exists $n \in \mathbb{N}$ such that for the interpolating polynomial $p_n(f) \in P_n$ with respect to $(x_i, f(x_i))$, $0 \leq i \leq n$, where $x_i = -1 + ih$, $h = 1/n$, there holds

$$\max_{-1 \leq x \leq +1} |f(x) - p_n(f)(x)| \geq M .$$

[Hinweis: Using the series expansion of f around some $a \in [-1, +1]$ show that for $k \rightarrow \infty$ the limes superior of $f^{(k)}(a)/k!$ is positive. Then, use the representation of the remainder in polynomial interpolation to estimate the interpolation error from below at $x = 1$.]

Delivery of the homework at latest on November 29, 2005. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.