



Department of Mathematics
University of Houston
Numerical Analysis I
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Numerical Analysis I (6th Homework Assignment)

Exercise 21 (*Machine accuracy*)

Let \mathbb{M} be a floating point arithmetic with basis $B \in \mathbb{N}, B \geq 2$, and mantissa length $L \in \mathbb{N}$. Moreover, let $r \in \mathbb{R}$ be an arbitrary reell number situated between two machine numbers of the same sign. Show that

$$\frac{|rd(r) - r|}{|r|} \leq \frac{1}{2} B^{1-L} .$$

Exercise 22 (*Error propagation for multiplication und division*)

Let \mathbf{M} be the set of machine numbers and let $\odot : \mathbf{M} \times \mathbf{M} \rightarrow \mathbf{M}$ and $\oslash : \mathbf{M} \times \mathbf{M} \rightarrow \mathbf{M}$ denote the multiplication and division in fixed point arithmetic, where $0 < eps \ll 1$ stands for the machine accuracy. Moreover, for $x, y \in \mathbf{R}$ let

$$\begin{aligned} rd(x) &= x(1 + \varepsilon_x) , \\ rd(y) &= y(1 + \varepsilon_y) . \end{aligned}$$

Verify the following estimates:

$$\begin{aligned} rd(x) \odot rd(y) &= (x \cdot y)(1 + \delta) , \\ |\delta| &\leq eps + (1 + eps)(|\varepsilon_x| + |\varepsilon_y| + |\varepsilon_x| |\varepsilon_y|) , \\ rd(x) \oslash rd(y) &= (x/y)(1 + \delta) , \\ |\delta| &\leq (|\varepsilon_x - \varepsilon_y| + eps(1 + eps))/(1 - eps) . \end{aligned}$$

Exercise 23 (*Arithmetic mean*)

Assume that $0 < a < b$ are floating point numbers with respect to the basis $B \in \mathbb{N}$, $B \geq 2$, and with mantissa length $L \in \mathbb{N}$, $L \geq 2$. Performing fixed point arithmetics with correct rounding, let m_1 and m_2 be the floating point numbers for $(a + b)/2$ and $a + (b - a)/2$, respectively.

- (i) For $B = 2$ show that $a \leq m_1 \leq b$.
- (ii) For $B = 10$ and $L = 2$ determine numbers a, b such that $m_1 \notin [a, b]$.
- (iii) Show that in any case $m_2 \in [a, b]$.

Delivery of the homework at latest on October 27, 2005. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.