Department of Mathematics
University of Houston
Numerical Analysis I
Dr. Ronald H.W. Hoppe

Numerical Analysis I
(1st Practical Homework Assignment)

Practical Exercise 1 (*Symmetric Toeplitz matrices*)

The autocorrelation function for the lag k of a time series $x(i)\ ,\ 1 \leq i \leq n$, is given by

$$R(k) \ = \ \frac{1}{n} \sum_{i=1}^{n-k} x(i)\ x(i+k) \quad , \quad 0 \leq k \leq n \ .$$

The coefficients $a_i\ ,\ 1 \leq i \leq n$, of the associated linear autoregressive process of order n

$$X_t \ = \ \sum_{i=1}^{n} a_i \ X_{t-i} \ + \ \varepsilon_t$$

(ε_t: white noise) are given by the solution $a = (a_1, ..., a_n)^T$ of the linear algebraic system

$$\underbrace{\begin{pmatrix} r_1 & r_2 & \cdot & \cdot & \cdot & r_n \\ r_2 & r_1 & \cdot & \cdot & \cdot & r_{n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ r_n & r_{n-1} & \cdot & \cdot & \cdot & r_1 \end{pmatrix}}_{R} \underbrace{\begin{pmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{pmatrix}}_{a} = \underbrace{\begin{pmatrix} -r_2 \\ -r_3 \\ \cdot \\ \cdot \\ \cdot \\ -r_{n+1} \end{pmatrix}}_{-r} ,$$

where R stands for the symmetric Toeplitz matrix with entries $r_k := R(k-1)\ ,\ 1 \leq k \leq n$, and $-r_k := -R(k-1)\ ,\ 2 \leq k \leq n+1$, are the components of the right-hand side $-r$.

a) Write an efficient code based on a direct method which reads the dimension $n \in \mathbb{N}$ and the vector $x \in \mathbb{R}^n$ from an input file and which has the solution vector $a \in \mathbb{R}^n$ as the output file. Take care of a possible termination in case the matrix R is not positive definite.
As time series choose monthly U.S unemployment rates (in percent) from January 1969 - July 2005.
(see http://www.economagic.com/em-cgi/data.exe/feddal/ru)

b) (voluntary part of the exercise)
A much more efficient algorithm for the solution of the linear system under consideration is the recursive Levinson-Durbin algorithm.
A C-program of the Levinson-Durbin algorithm can be found in th Numerical Recipes.
(http://www.ulib.org/webRoot/Books/Numerical_Recipes/bookcpdf/c2-8.pdf)

In MATLAB, the function levinson is available within the MATLAB toolbox.

Solve the linear system from part a) by the Levinson-Durbin algorithm.

Delivery of an executable program at latest on September 27, 2005. Submit a program listing and the input and output data for at least two selected test cases.