Department of Mathematics, University of Houston
Dr. Ronald H.W. Hoppe
Optimization Theory, Fall 2006

Optimization Theory
(2nd Homework Assignment)

<u>**Exercise 4**</u> (*Simplex Method*)

Consider the linear problem

$$\begin{array}{rrcl}
\text{minimize} & 3x_1 - x_2 - x_3 & & \\
\text{subject to} & x_1 + x_2 + x_3 & \leq & 4 \; , \\
 & x_1 & \leq & 2 \; , \\
 & x_3 & \leq & 3 \; , \\
 & 3x_2 + x_3 & \leq & 6 \; , \\
 & x_1, x_2, x_3 & \geq & 0 \; .
\end{array}$$

In standard form $Ax = b, x \geq 0$, the feasible set can be written as follows

$$\begin{pmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 3 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \\ 6 \\ 0 \end{pmatrix} \; , \; x \geq 0 \; .$$

(i) Is the index vector $J_1 = \{1, 2, 3, 5, 8\}$ a basis? Identify the associated vertex of the polyhedron representing the feasible set.

(ii) Is the index vector $J_1 = \{1, 2, 3, 5, 8\}$ a feasible basis? If so, is it non-degenerate?

(iii) Is the index vector $J_2 = \{4, 2, 3, 5, 8\}$ a basis? Identify the associated vertex of the polyhedron representing the feasible set.

(iv) Is the index vector $J_2 = \{4, 2, 3, 5, 8\}$ a feasible basis? If so, is it non-degenerate?

(v) Consider the bases $J_1 = (1, 2, 3, 5, 8)$ and $J_2 = (4, 2, 3, 5, 8)$. Are the optimality conditions satisfied?

(vi) Use the simplex method to compute the solution of the LP starting from the feasible basis J_1.

12 Points

Delivery of the homework at latest on Sept. 6, 2006, 04:00 pm. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class