Department of Mathematics, University of Houston
Dr. Ronald H.W. Hoppe
Optimization Theory, Fall 2006

Optimization Theory
(4th Homework Assignment)

Exercise 6 (*Unbounded Solution*)

Consider the linear program

$$\begin{aligned} &\text{minimize} \quad c^T x \\ &\text{subject to } Ax = b \ , \ x \geq 0 \quad , \end{aligned}$$

where $c, x \in \mathbb{R}^n, b \in \mathbb{R}^m$, and $A \in \mathbb{R}^{m \times n}$. Assume that $d \in \mathbb{R}^n$ is a direction for which

$$c^T d < 0 \ , \ Ad = 0 \ , \ d_i \geq 0 \text{ for each } i \text{ with } x_i = 0 \ .$$

Show that if x is a feasible vector for the LP, then $x + \alpha d$ is also feasible and

$$c^T(x + \alpha d) \ < \ c^T x$$

for all sufficiently small $\alpha > 0$. Deduce that if such a vector d exists, the objective functional $c^T x$ is unbounded from below on its feasible region and hence, the LP has no finite solution.

4 Points

Exercise 7 (*LP in Nonstandard Form*)

Consider the following linear program in nonstandard form

$$\begin{aligned} &\text{minimize} \quad c^T w + d^T z \quad \text{over } (w, z) \\ &\text{subject to } A_{11} w + A_{12} z \geq b_1 \ , \ A_{21} w + A_{22} z = b_2 \ , \ w \geq 0 \quad . \end{aligned}$$

(i) By splitting and introducing slack variables, convert this problem to standard form.

(ii) Write down the dual problem for the standard form version, and express it in a cleaner form by eliminating the slack variables.

4 Points

Exercise 8 (*Sufficiency of the KKT conditions and optimal objective values*)

Assume that (x, λ, s) satisfies the KKT conditions from Theorem 1.2.

(i) Show that any primal feasible vector $\bar{x}$ has $c^T\bar{x} \geq c^T x$.

(ii) Show that any dual feasible vector $(\bar{\lambda}, \bar{s})$ has $b^T\bar{\lambda} \leq b^T\lambda$.

(iii) Show that $c^T x = b^T\lambda$, i.e., the optimal objective values of the primal and dual problem are the same.

6 Points

Delivery of the homework at latest on Sept. 20, 2006, 04:00 pm. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class