Department of Mathematics, University of Houston
Dr. Ronald H.W. Hoppe
Optimization Theory, Fall 2006

Optimization Theory
(5th Homework Assignment)

Exercise 9 (*Primal Central Path*)

Given $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c \in \mathbb{R}^n$, the primal central path $\mathcal{C}_P$ is the trajectory of all points $x(\tau) \in \mathbb{R}^n, x(\tau) > 0, \tau > 0$, which solve the nonlinear system

$$(*) \qquad f_P(x, \lambda, \tau) \; := \; \begin{pmatrix} Ax \; - \; b \\ \tau D_x^{-1} e \; + \; A^T \lambda \; - \; c \end{pmatrix} \; = \; \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

for some $\lambda = \lambda(\tau) \in \mathbb{R}^m$, where $D_x := \text{diag}(x_1, ..., x_n)$ and $e = (1, ..., 1)^T$. Consider the specific example

$$A \; = \; \begin{pmatrix} -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \quad , \quad b \; = \; (2,4)^T \quad , \quad c \; = \; (1,2,0,0)^T \; .$$

(i) Specify the nonlinear system defining the associated primal central path.

(ii) Show that the elimination of x_3, x_4 and λ_1, λ_2 from the nonlinear system in (i) gives rise to a reduced parameter dependent nonlinear system in x_1, x_2,

$$f(x_1, x_2, \tau) \; = \; 0.$$

Compute this nonlinear system.

(iii) Show by the implicit function theorem that for every $\tau > 0$ the nonlinear system from (ii) has a unique solution $(x_1(\tau), x_2(\tau))$ in a vicinity of $(1,3)^T$.

(iv) If $x(\tau) = (x_1(\tau), x_2(\tau))^T$ is the solution of the nonlinear system from (ii), show that

$$\lim_{\tau \to 0} x(\tau) \; = \; (1,3)^T \; .$$

What does this result tell you?

8 Points

Exercise 10 (*Modified logarithmic potential function*)

Karmarkar's celebrated interior point algorithm focuses solely on the primal problem measuring progress towards optimality by means of a logarithmic potential function different from the one in the definition of the primal central path. Soon after Karmarkar, Renegar devised an algorithm that uses Newton's method in conjunction with another logarithmic function, namely

$$(**) \qquad \text{minimize} \quad -n\log(Z - c^T x) - \sum_{i=1}^{n} \log(x_i) \quad \text{over } x \in \mathbb{R}^n$$
$$\text{subject to } Ax = b \ , \ x > 0 \ , \ c^T x < Z \ ,$$

where Z is an upper bound on the optimal value Z^* of $c^T x$.

(i) Write down the KKT conditions for the minimization problem (**).

(ii) By relating the KKT conditions from (i) to the conditions defining the primal central path, show that for a certain value of τ the solution of (**) is situated on the primal central path.

4 Points

Delivery of the homework at latest on Sept. 27, 2006, 04:00 pm. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class