Department of Mathematics, University of Houston
Dr. Ronald H.W. Hoppe
Optimization Theory, Fall 2006

Optimization Theory
(8th Homework Assignment)

Exercise 17 *Quadratic Programming Problem*

Consider the QP

$$\begin{aligned} &\text{minimize} \quad 3x_1^2 + 2x_1x_2 + x_1x_3 + 2.5x_2^2 + 2x_2x_3 + 2x_3^2 - 8x_1 - 3x_2 - 3x_3 \\ &\text{over} \quad x = (x_1, x_2)^T \in \mathbb{R}^2 \\ &\text{subject to} \quad x_1 + x_3 = 3 \quad , \quad x_2 + x_3 = 0 \ . \end{aligned}$$

Identify the matrices $B \in \mathbb{R}^{3\times 3}$, $A \in \mathbb{R}^{2\times 3}$ and the vectors $b \in \mathbb{R}^3$, $c \in \mathbb{R}^2$ of the KKT system and compute its solution x^* , λ^*. Moreover, determine the null-space basis matrix Z. **4 points**

Exercise 18 *Null-Space Approach*

Solve the QP problem of Exercise 17 by the null-space approach. In particular, specify the matrices Y and Z and compute w_Y and w_Z. **4 points**

Exercise 19 *Schur complement*

Suppose that $B \in \mathbb{R}^{n\times n}$ is symmetric positive definite, $A \in \mathbb{R}^{m\times n}, m \leq n$, such that $\text{rank}(A) = m$ and let $S = -AB^{-1}A^T$ be the Schur complement associated with the KKT-matrix. Show that $-S$ is symmetric positive definite on $\mathbb{R}^m$.
4 points

Delivery of the homework at latest on November 1, 2006, 04:00 pm. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class