University of Houston
Department of Mathematics
Ronald H.W. Hoppe
Numerical Methods for Option Pricing

Numerical Methods for Option Pricing (Homework 2)

Exercise 4 (*Binomial Method*)

Consider a binomial model for the price $S_n, n \in \{0,1,2,3\}$, of a stock with $S_0 = 100$ under the assumption that at each trading time the price either goes up or down by 10% and that the risk-free interest rate is 5%. Consider a European put option with strike $K = 110$ and maturity date $T = 3$.

(i) Compute the risk-neutral probability p.

(ii) Compute the price P_0 of the put option.

(iii) Specify the duplicating portfolio.

6 Points

Exercise 5 (*Nonlinear system in binomial method*)

As has been shown in class, in the binomial method for the computation of the price of European or American put and call options, the parameters u (up), d (down), and p (probability) are chosen such that the expectations and variances of the time-discrete model are equal to those of the time-continuous model. This leads to the nonlinear system

$$\begin{aligned} \exp(r\Delta t) &= pu + (1-p)d \,, \\ \exp((2r+\sigma^2)\Delta t) &= pu^2 + (1-p)d^2 \,, \\ ud &= 1 \,. \end{aligned}$$

Show that the solution of this nonlinear system is given by

$$u = \beta + \sqrt{\beta^2 - 1} \quad , \quad d = \beta - \sqrt{\beta^2 - 1} \,,$$
$$p = \frac{\exp(r\Delta t) - d}{u - d} \quad , \quad \text{where} \quad \beta = \frac{1}{2}\left(\exp(-r\Delta t) + \exp((r+\sigma^2)\Delta t)\right) \,.$$

4 Points

Exercise 6 (*Implementation of Binomial Method*)

Use the MATLAB program as introduced in class (see also reverse side) to compute the prices of the following financial derivatives:

(i) A European put option with $K = 100$, $S_0 = 103$, $r = 0.04$, $\sigma = 0.3$.

(ii) A European call option with $K = 100$, $S_0 = 95$, $r = 0.1$, $\sigma = 0.2$.

(iii) Compute both for (i) and (ii) the number of time steps for which the relative error (compared to the Black-Scholes price) is less than 0.01%.

6 Points

Exercises 4 and 5 are due on September 19, 2007. Exercise 6 is due on November 19, 2007. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class

The following MATLAB program computes the price of a European put option according to the binomial method. The input parameters ∘ have to be specified by the user. The commands will be sequentially compiled and executed by the MATLAB interpreter. For appropriate outputs see the MATLAB handbook.

```
%Input parameters
K = ∘; S0 = ∘; r = ∘; sigma = ∘; T = ∘; N = ∘;
% Computation of u, d, p
beta = 0.5 * (exp(−r * dt) + exp((r + sigma^2) * dt));
u = beta + sqrt(beta^2 − 1);
d = 1/u;
p = (exp(r * dt) − d)/(u − d);
%First step
for j = 1 : N + 1
    S(j, N + 1) = S0 * u^(j − 1) * d^(N − j + 1)
end
%Second step
for j = 1 : N + 1
    V(j, N + 1) = max(K − S(j, N + 1), 0);
end
%Third step
e = exp(−r * dt);
for i = N : −1 : 1
    for j = 1 : i
        V(j, i) = e * (p * V(j + 1, i + 1) + (1 − p) * V(j, i + 1));
    end
end
%Output
fprintf('V(%f, 0) = %f \ n', S0, V(1, 1))
```