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Numerical Methods for Option Pricing

Numerical Methods for Option Pricing (Homework 3)

Exercise 7 (*Linear Diffusion Equation I*)

Given $u_0 : \mathbb{R} \to \mathbb{R}$, show that the function

$$u(x,t) \;=\; \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{+\infty} u_0(s)\, \exp(-(x-s)^2/4t)\; ds \quad , \quad x \in \mathbb{R}\,,\; t > 0\,,$$

solves the linear diffusion equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} \;=\; 0 \quad , \quad x \in \mathbb{R}\,,\; t > 0\,,$$

and satisfies

$$\lim_{t \to 0} u(x,t) \;=\; u_0(x) \quad , \quad x \in \mathbb{R}\,.$$

4 Points

Exercise 8 (*Linear Diffusion Equation II*)

Given $r > 0, \sigma \geq 0$, let $\kappa := 2r/\sigma^2$ and

$$\Phi(x) \;=\; \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-s^2/2)\; ds\,,$$

$$d_{1/2} \;=\; \frac{\ln(S/K) \;+\; (r \pm \sigma^2/2)(T-t)}{\sigma\,\sqrt{T-t}}\,.$$

Show that there holds

$$\frac{1}{\sqrt{2\pi}} \int_{-x/\sqrt{2\tau}}^{\infty} \exp(\frac{1}{2}(\kappa \pm 1)(x + y\sqrt{2\tau}))\, \exp(-y^2/2)\; dy \;=$$

$$=\; \exp(\frac{1}{2}(\kappa \pm 1)x + \frac{1}{4}(\kappa \pm 1)^2\tau)\; \Phi(d_{1/2})\,.$$

4 Points

Exercise 9 (*Black-Scholes Formula for European Calls and Puts*)

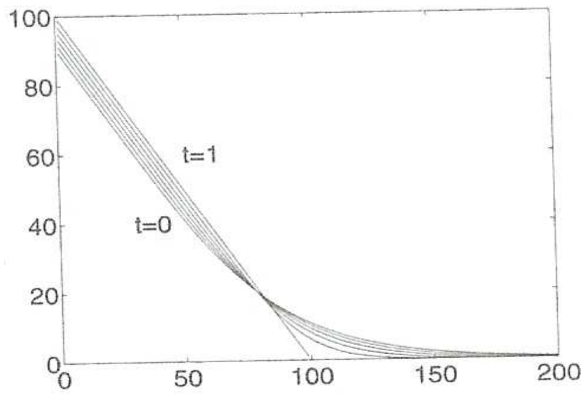
As has been shown in class, the Black-Scholes formula for a European call is

$$V(S,t) \;=\; S\;\Phi(d_1) \;-\; K\;\exp(-r(T-t))\;\Phi(d_2)\;,$$

whereas for a European put it takes the form

$$V(S,t) \;=\; K\;\exp(-r(T-t))\;\Phi(-d_2) \;-\; S\;\Phi(-d_1)$$

with Φ and $d_\nu, 1 \le \nu \le 2$, as given in Exercise 8.
Write a MATLAB-program which computes the values of a European call and of a European put for $K = 100, T = 1, r = 0.1, \sigma = 0.4$ and displays them as a function of S for $t = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ as shown in the following graphics in case of the put-option:



6 Points

Exercises 7 and 8 are due on October 3, 2007. Exercise 9 is due on November 19, 2007. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class