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Numerical Methods for Option Pricing

Numerical Methods for Option Pricing (Homework 4)

Exercise 10 (*Continuous dividends*)

In case of options on indices (e.g., on the Dow Jones etc.), dividends are paid throughout the entire year. Therefore, the assumption of continuous dividends represents a simple approach to consider dividends in the Black-Scholes model.

(i) Assume that the dividend is proportional to the value S of the asset and that after the time interval Δt the dividend

$$D \; S \; \Delta t \; ,$$

is paid, where D stands for the proportionality factor. In order to exclude arbitrage, the value of the asset must be reduced accordingly. Assuming that the value of the asset satisfies a geometric Brownian motion with μ, σ, determine the modified Itô process for the value of asset.

(ii) Consider a portfolio consisting of c_1 bonds B with risk-free interest rate $r > 0$, c_2 stocks S, and a sold option $V = V(S,t)$. Determine the associated Itô process under the assumption of a self-financing portfolio. Apply Itô's formula to V and show that a risk-free portfolio satisfies the partial differential equation

$$\frac{\partial V}{\partial t} \; + \; \frac{1}{2} \, \sigma^2 \, S^2 \, \frac{\partial^2 V}{\partial S^2} \; + \; (r - D) \, S \, \frac{\partial V}{\partial S} \; - \; r \, V \; = \; 0 \; .$$

What is the appropriate choice of c_2?

(iii) In case of continuous dividends and European options, show that the put-call parity is of the form

$$P_t \; - \; C_t \; = \; K \, \exp(-r(T-t)) \; - \; S \, \exp(-D(T-t)) \; .$$

What are the boundary conditions for the partial differential equations from (ii), if $V = C$ is a European call?

6 Points

Exercise 11 (*Greeks I*)

(i) Using the Black-Scholes formula for the price V of a European call, compute the sensitivities

$$\Delta \; = \; \frac{\partial V}{\partial S} \quad , \quad \Gamma \; = \; \frac{\partial^2 V}{\partial S^2} \; .$$

In order to do so, show first that the distribution function Φ of the standard normal distribution satisfies

$$S\ \Phi'(d_1) \ = \ K\ \exp(-r(T-t))\ \Phi'(d_2)\ .$$

(ii) Deduce from (i) that the price V is a convex function. Use this property to show that the fraction of the bond

$$c_1(t) \ = \ \frac{1}{B(t)}\ (V(S(t),t) - S(t)\ \frac{\partial V}{\partial S})$$

is a strictly negative function.
[Hint: Use Taylor expansion of $V(0,t) = 0$ around $V(S,t)$.]
(iii) Further, compute the remaining Greeks

$$\theta \ = \ \frac{\partial V}{\partial t} \quad , \quad \rho \ = \ \frac{\partial V}{\partial r} \quad , \quad \kappa \ = \ \frac{\partial V}{\partial \sigma}\ .$$

6 Points

<u>**Exercise 12**</u> (*Greeks II*)

Write a MATLAB-program *greeks.m* which computes the *greeks* Delta, Gamma, Vega, Theta, and Rho of a European call and displays them in a graphics at $t = 0$ (dotted line), $t = 0.4$ (dashed line), and $t = 0.8$ (straight line) for

$$K \ = \ 100 \quad , \quad T \ = \ 1 \quad , \quad r \ = \ 0.1 \quad , \quad \sigma \ = \ 0.4\ .$$

6 Points

Exercises 10 and 11 are due on October 17, 2007. Exercise 12 is due on November 19, 2007. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class