University of Houston
Department of Mathematics
Ronald H.W. Hoppe
Numerical Methods for Option Pricing

Numerical Methods for Option Pricing (Homework 5)

Exercise 13 (*Linear congruential random number generators*)

A linear congruential generator for pseudo random numbers has the form

$$X_i \ := \ aX_{i-1} \ + \ b \bmod M \quad , \quad i \geq 1 \ ,$$

where a is a fixed multiplicator, b stands for the increment, M is the modulus, and X_0 is an arbitrary start value.

(i) Why does the algorithm provide periodic sequences of pseudo random numbers with a period of at most M?

(ii) The numbers a, b, M can be chosen such that the resulting sequences have the maximal period M. If, however, $b = 0$ (as, for instance, in the generator RANDU), the period is always less than M. Why?

4 Points

Exercise 14 (*Distribution of two-dimensional random vectors*)

Assume that the sequence of pseudo random numbers $U_i \in [0,1), i \in \mathbb{N}_0$, is given by the linear congruential method

$$X_i \ := \ aX_{i-1} \ + \ b \bmod M \quad , \quad U_i \ := \ X_i/M \ ,$$

where $a = 1229$, $b = 1$, $M = 2048$, and $X_0 \in Z$ is an arbitrarily chosen start value.

Show that the random vectors (U_{i-1}, U_i) are located on at most six different straight lines within the unit square $[0,1)^2$.

4 Points

Hint: Observe $-1 + 5a \equiv 0 \bmod M$.

Exercise 15 (*Distribution of three-dimensional random vectors*)

Consider the pseudo random number generator RANDU

$$X_i \ := \ aX_{i-1} \bmod M \quad , \quad U_i \ := \ X_i/M \ ,$$

where $a = 2^{16} + 3$ and $M = 2^{31}$.

Show that for an arbitrarily chosen $X_0 \in Z$ the values

$$U_{i+2} \ - \ 6U_{i+1} \ + \ 9U_i \quad , \quad i \in \mathbb{N}_0$$

are entire numbers and describe the distribution of (U_i, U_{i+1}, U_{i+2}) in the unit cube $[0,1)^3$.

4 Points

<u>Exercise 16</u> (*Marsaglia's polar algorithm*)

Implement the following MATLAB-program *polar.m* which realizes Marsaglia's polar algorithm for the generation of N^2 standard normally distributed random numbers (W_1, W_2). Observe that the algorithm provides approximately $0.79 \cdot N^2$ random numbers, since all random numbers (W_1, W_2) with $W_1^2 + W_2^2 \geq 1$ are rejected. The command *hist(Z,[a:b:c])* generates a histogram of Z with respect to the intervals $[a + ih, a + (i+1)h], 0 \leq i \leq (c-a)/b - 1, h := (c-a)/b$.

```
rand('state',1); N = 200000;
U1 = rand(N,1); U2 = rand(N,1);
% Compute two vectors of in [−1,+1] equally distributed random variables
W1 = 2*U1 - ones(N,1);
W2 = 2*U2 - ones(N,1);
% Compute the set I of indices for which W1^2 + W2^2 < 1
W = W1.^2 + W2.^2;
I = find(W<1);
% Compute two standard normally distributed random variables
V = V(I);
Z1 = V1(I).*sqrt(-2.*log(V)./V);
Z2 = V2(I).*sqrt(-2.*log(V)./V);
% Generation of histograms
hist(Z1,[0.0:0.05:1.0);
hist(Z2,[0.0:0.05:1.0)
```

4 Points

Exercises 13,14 and 15 are due on October 31, 2007. Exercise 16 is due on November 19, 2007. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class