Department of Mathematics
University of Houston
Numerical Analysis I
Dr. Ronald H.W. Hoppe

Numerical Mathematics I
(8. Homework Assignment)

Exercise 27 (*Regula falsi of higher order*)

Assume that $f \in C^1([a,c)), 0 < a < c$, has a simple zero $x^* \in [a,c]$ and that $f(a)f(c) < 0$. Let b such that $a < b < c$ and consider the quadratic polynomial $p \in P_2(\mathbb{R})$ interpolating f in a, b, c, i.e. $p(z) = f(z), z \in \{a, b, c\}$.

(i) Show that p has exactly one zero in $[a, c]$.

(ii) Denote by

$$\text{REAL zero}(a, b, c, f)$$

the function which computes the zero of p for given a, b, c. Using this function, construct an iterative algorithm which provides an approximation of x^* up to machine accuracy eps.

Exercise 28 (*Newton's method for the computation of the m-th root*)

(i) For $m \in \mathbb{N}$, the positive m-th root $x^* > 0$ of $a > 0$ is the solution of the equation

$$f(x) \;=\; 0 \quad \text{where } f(x) \;:=\; x^m \;-\; a \;.$$

Assume $m \geq 2$ and determine the Newton iteration

$$x^{(k+1)} \;=\; \Phi(x^{(k)})$$

for the computation of x^*.

(ii) Let f be a scalar real-valued function with zero x^* and denote by $x^{(k)}, k \in \mathbb{N}$, the iterates obtained by Newton's method. Show that there holds

$$x^* \;-\; x^{(k+1)} \;=\; -\,\frac{1}{2}\,\frac{f''(x^{(k)} + \theta(x^+ - x^{(k)}))}{f'(x^{(k)})}\,(x^* - x^{(k)})^2 \;,$$

where $0 \leq \theta \leq 1$.

(iii) For the particular function f from part (i) compute the asymptotic error constant

$$C \;:=\; \frac{1}{2}\,|\frac{f''(x^*)}{f'(x^*)}| \;.$$

What is its meaning with regard to the convergence of Newton's method close to the zero x^*?

<u>Exercise 29</u> (*Iterative procedure for the division*)

For the implementation of the division, modern processors use Newton's method with a fixed number of iterations. In the following assume double precision IEEE-arithmetics, i.e., basis $B = 2$ and mantissa length $L = 53$.

(i) Construct a Newton iteration for the computation of $1/a, a > 0$, that only uses addition/subtraction and multiplication.

(ii) Let a be a normalized floating point number $a = \bar{a}2^E$ with mantissa $\bar{a} \in [0.5, 1)$. Using the Newton iteration from part (i) with $x^{(0)} = 1$ as an initial iterate, construct an algorithm for the efficient computation of $1/a$.

(iii) A requirement for the division is that the resulting relative rounding error is below the machine accuracy. Using $x^{(0)} = 1$ as initial iterate, how many Newton steps are necessary to achieve this goal, provided rounding errors during the iteration can be neglected?

Delivery of the homework at latest on November 10, 2005. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.