University of Houston
Department of Mathematics
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Numerical Analysis II, Spring 2010

Numerical Analysis II
(Homework 3)

Exercise 7 (*Maximum consistency order of s-stage Runge Kutta methods*)

Show that the application of s-stage Runge Kutta methods to the initial-value problem

$$y'(x) \; = \; \lambda \, y(x) \quad , \quad y(x_0) \; = \; y_0$$

gives rise to increment functions of the form

$$k_j(x_k, y_k) \; = \; p_j(\lambda h) \, y_k \quad , \quad 1 \leq j \leq s \; ,$$

where p_j are polynomials of degree at most j. Use this result for the determination of the maximum consistency order of s-stage Runge Kutta methods.

6 Points

Exercise 8 (*Runge Kutta methods without memory*)

(i) Compute all Runge Kutta methods of order 2 having the Butcher scheme

$$\begin{array}{r|ccc} c_1 = 0 & & & \\ c_2 & c_2 & & \\ c_3 & 0 & c_3 & \\ \hline & 0 & 0 & 1 \end{array}$$

(ii) In algorithmic notation derive an efficient implementation with respect to storage.

(iii) Compute the polynomials p_j , $1 \leq j \leq 3$, as introduced in Exercise 7.

6 Points

Exercise 9 (*Asymptotic expansion for the explicit Euler method*)

Gragg's theorem tells us that the coefficients in the asymptotic expansion of the global discretization error are solutions of inhomogeneous linear initial-value problems. These initial-value problems should be derived for a specific problem: The discretization of the initial-value problem $y'(x) \; = \; y(x)$, $y(a) \; = \; \alpha$ by the explicit Euler scheme with constant step size h gives rise to the difference equation

$$y_h(x+h) \; = \; y_h(x) \; + \; h \, y_h(x) \quad , \quad y_h(a) \; = \; \alpha \; .$$

Consider the asymptotic expansion

$$y_h(x) \;=\; \sum_{k=0}^{\infty} e_k(x)\; h^k$$

and compute the staggered system of differential equations for the determination of the coefficients $e_k, k \in \mathbb{N}_0$, as well as the initial conditions. Compute e_0 and e_1 explicitly.

6 Points

Delivery of the homework at latest on March 4, 2010. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.